

# An Investigation of How Individuals Revise Their Beliefs

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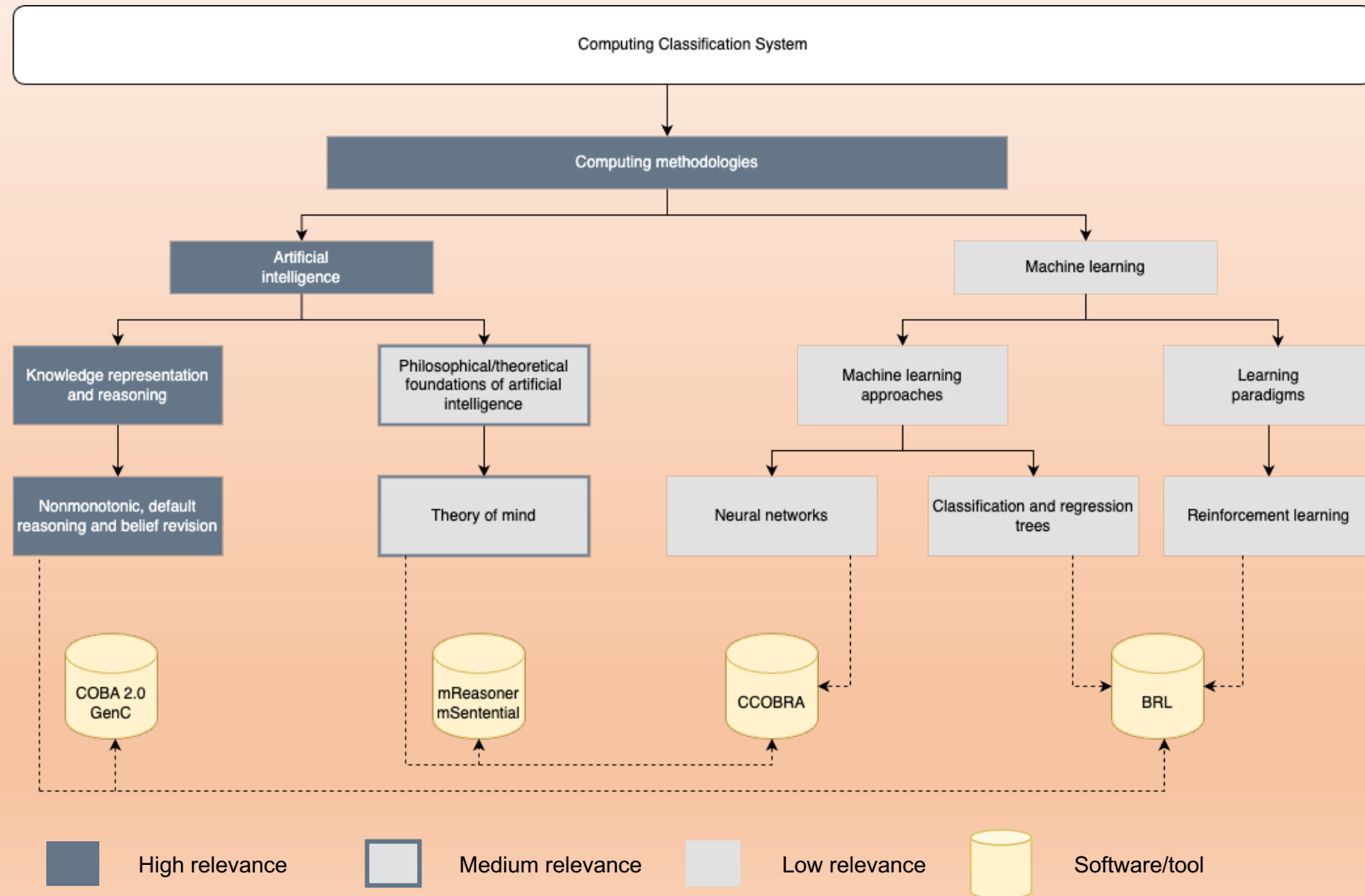
# Talk outline

1. Conceptual framework
2. Background
3. AGM vs KM revision
4. Cognitive modelling
5. Predictive modelling
6. Software tools
7. Experiments
8. Contributions



# Conceptual framework

<https://dl.acm.org/ccs>, 2012

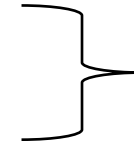




# Background

## Example

- I believe there are 3 cups in the dining room
- each cup is either on the dining table or not



***knowledge base, belief base;***  
*contains statements about the world*





# Background

## Example

- I believe there are 3 cups in the dining room
- each cup is either on the dining table or not



*single interpretation,*  
*possible world* }

<i><b>a</b></i>	<i><b>b</b></i>	<i><b>c</b></i>
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F



# Background

## Example

- I believe there are 3 cups in the dining room
- each cup is either on the dining table or not

After sending a robot into the dining room, it reports:

- cup *a* is not on the table



*models of the world,  
after revision*

<i><b>a</b></i>	<i><b>b</b></i>	<i><b>c</b></i>
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F



# Background

## Example

- I believe there are 3 cups in the dining room
- each cup is either on the dining table or not

After sending a robot into the dining room, it reports:

- cup *a* is not on the table
- cup *b* is on the table



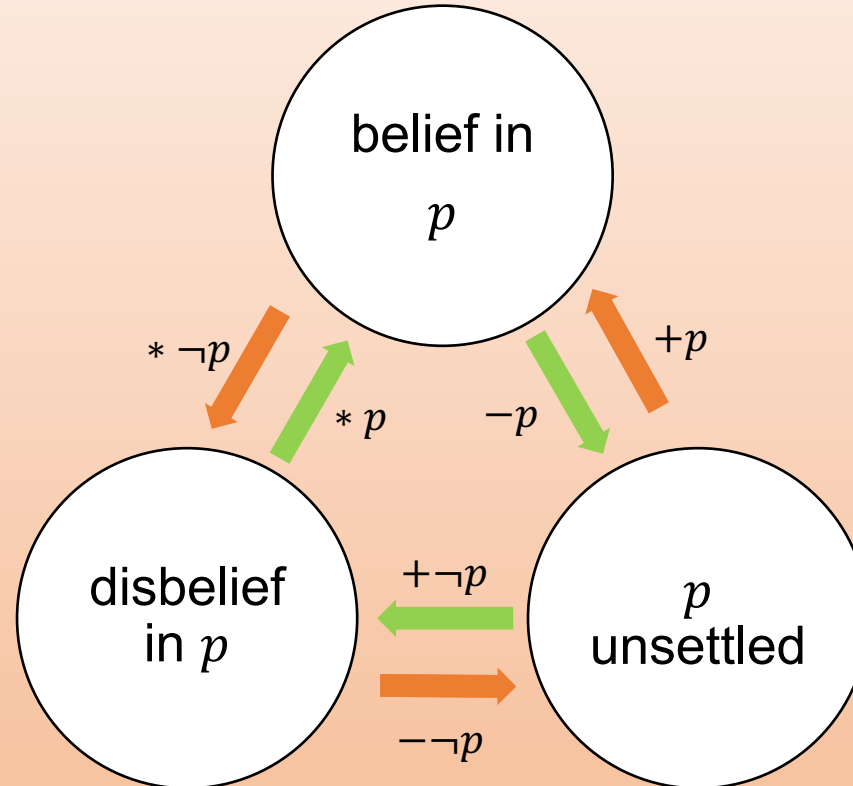
*models of the world,  
after revision again*

<i><b>a</b></i>	<i><b>b</b></i>	<i><b>c</b></i>
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F



# Background

Fermé and Hansson, 2018



+ expansion  
- contraction  
\* revision  
 $\neg$  negation

Remark: The six types of belief change hold under the assumption that  $\nvdash p$  and  $\nvdash \neg p$

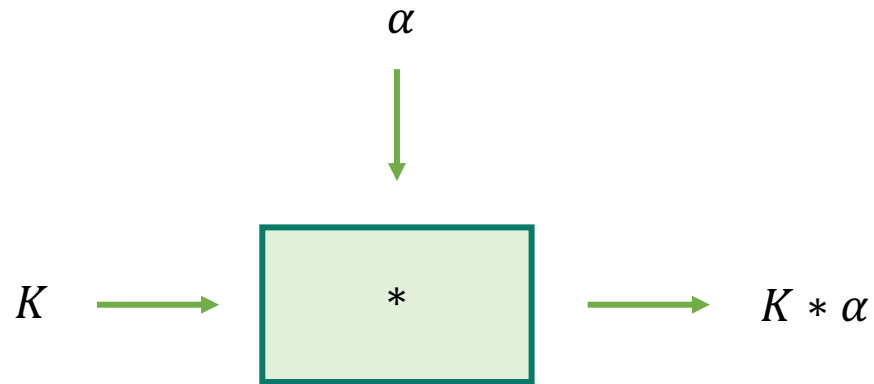


# AGM belief revision

Alchourrón, Gärdenfors and Makinson, 1985

## Approach

- considers different pieces of information that refer to the same setting
- supposes a belief set, a deductively closed set of formulae to represent beliefs
- revision is performed by a binary operator



- defines postulates that a revision operator must satisfy to be considered 'rational'

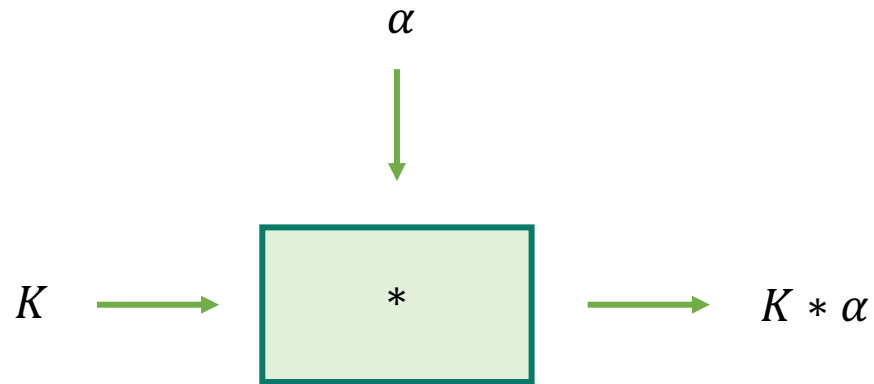


# AGM belief revision

Alchourrón, Gärdenfors and Makinson, 1985

## Drawbacks

- forces a revision operator to be too conservative
- discards all information about the old belief set if revising with an inconsistent formula



- does not consider dynamic situations (out of scope for this work)



# AGM belief revision

Alchourrón, Gärdenfors and Makinson, 1985

- R1.  $K = C_n(K)$  and  $K * \alpha = C_n(K * \alpha)$
- R2. If  $K * \alpha \models \beta$  then  $K + \alpha \models \beta$
- R3. If  $K \not\models \neg\alpha$  then (if  $K + \alpha \models \beta$  then  $K * \alpha \models \beta$ )
- R4.  $\alpha \in K * \alpha$
- R5. If  $\alpha \equiv \beta$  then  $K * \alpha \models \gamma$  iff  $K * \beta \models \gamma$
- R6. If  $\alpha \not\models \perp$  then  $K * \alpha \not\models \perp$
- R7. If  $K * (\alpha \wedge \beta) \models \gamma$  then  $(K * \alpha) + \beta \models \gamma$
- R8. If  $K * \alpha \not\models \neg\beta$  then (if  $(K * \alpha) + \beta \models \gamma$  then  $K * (\alpha \wedge \beta) \models \gamma$ )

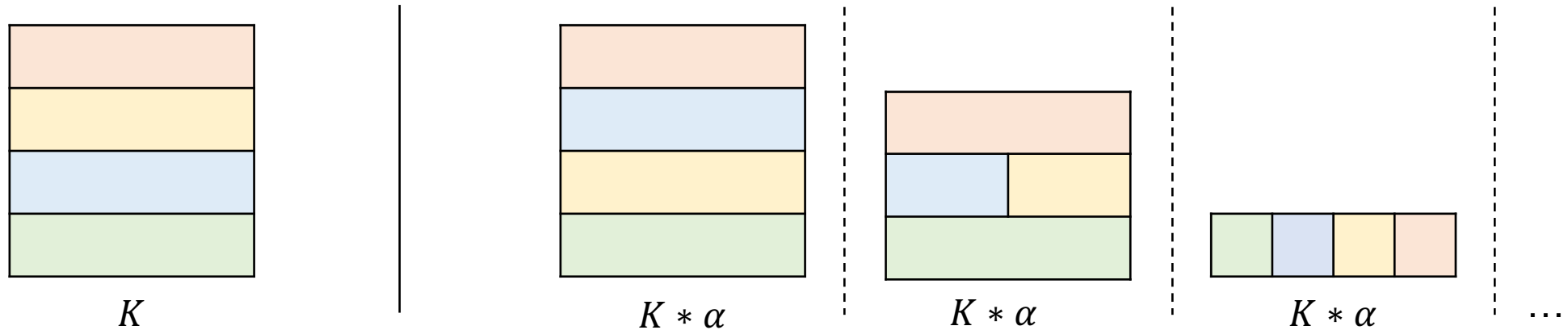


# KM belief revision

Katsuno and Mendelzon, 1991

## Approach

- supposes a belief base, a set of formulae, not deductively closed
- uses possible worlds semantics (a pre-order amongst models) to model belief revision
- satisfies the AGM postulates lifted to this setting



- leaves the original belief base unchanged



# KM belief revision

Katsuno and Mendelzon, 1991

- U1.  $K \diamond \alpha \models \alpha$
- U2. If  $K \models \alpha$  then  $K \diamond \alpha$  iff  $K$
- U3. If both  $K$  and  $\alpha$  is satisfiable then  $K \diamond \alpha$  is satisfiable
- U4. If  $K_1$  iff  $K_2$  and  $\alpha$  iff  $\beta$  then  $K_1 \diamond \alpha$  iff  $K_2 \diamond \beta$
- U5.  $(K \diamond \alpha) \wedge \gamma \models K \diamond (\alpha \wedge \gamma)$
- U6. If  $K \diamond \alpha \models \beta$  and  $K \diamond \beta \models \alpha$  then  $K \diamond \alpha$  iff  $K \diamond \beta$
- U7. If  $K$  is complete then  $(K \diamond \alpha) \wedge (K \diamond \beta) \models K \diamond (\alpha \vee \beta)$
- U8.  $(K_1 \vee K_2) \diamond \alpha$  iff  $(K_1 \diamond \alpha) \vee (K_2 \diamond \alpha)$
- U9. If  $K$  is complete and  $(K \diamond \alpha) \wedge \gamma$  is satisfiable  
then  $K \diamond (\alpha \wedge \gamma) \models (K \diamond \alpha) \wedge \gamma$





## **Do Humans Find Postulates of Belief Change Plausible?**

[Journal of Applied Logic, 2023]

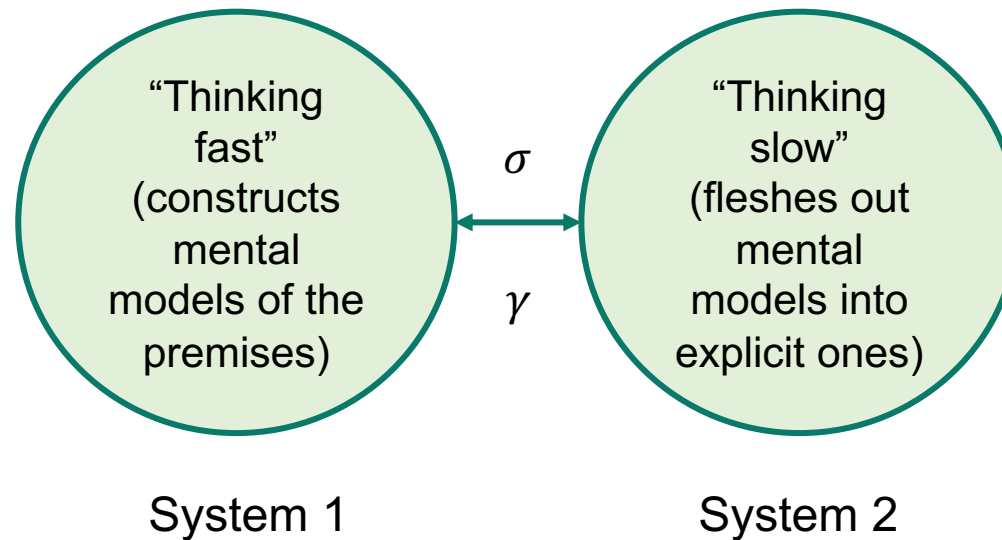


# Cognitive modelling

Khemlani et al. 2018; <https://www.modeltheory.org/models/>

## mSentential

- Common Lisp program
- propositional reasoner
- implements two systems of thinking, intuitive and deliberative



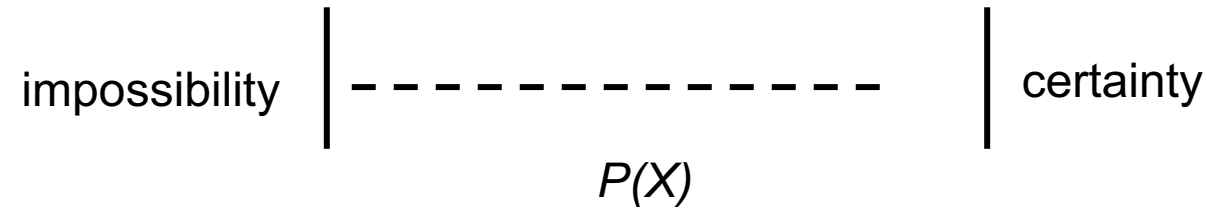


# Cognitive modelling

Byrne and Johnson-Laird, 2020; <https://www.modeltheory.org/models/>

## mReasoner

- updated version of mSentential
- assumes models have an underlying iconic structure... Johnson-Laird, 1983



- iterates over loops to form the intuitive averages of the two values of an icon
- pushes the values towards each other
- the result can be mapped to a numerical estimate of a probability



# Predictive modelling

## Motivation

- new paradigm to reverse-engineer cognitive models, solvers for reasoning
- sample data does not often generalise to individual data
- parameters of cognitive models can be tuned to uncover individual data





# Software tools

	COBA 2.0	CCOBRA	GenC	BRL
Implementation	interactive Java applet	CLI, python	CLI, C++, OpenMP	CLI, C++
Use	solver for AGM revision via consistency-based propositional reasoning	predicts individual responses based on task type and response type using cognitive models, NNs	solver for AGM revision using class of parameterised difference (PD) operators	computes AGM revision models and predicts future revisions
Input	belief base, sentence	labelled tasks, response types, and responses	belief base, sentence, ordering	history of beliefs and belief states (0-1)
Performance	single-shot AGM revision, slow	matches, exceeds frequency models	single-shot AGM revision, fast	multiple revision, fast
Citation	Delgrande et al. 2007	Riesterer et al. 2019, 2020	Hunter and Agapayev, 2021	Hunter and Boyarinov, 2022



# Experiments



Bob

...	?
2	?
1	?
0	?

*old beliefs*

I believe that  
penguins are  
flightless winged  
birds

*new information*

Did you know  
that magic  
penguins fly?



Alice

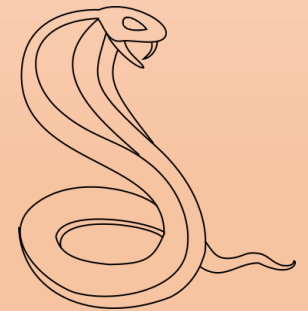
*new beliefs*

Strange! Let me  
revise...



Bob

...	?
2	?
1	?
0	?





# Contributions

## Current

- design survey of model-based belief revision with human subjects on real examples
- collect data

## Next

- benchmark revision data on cognitive, predictive models
- do predictive analysis of individual human belief revision



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<https://claykbaker.wordpress.com>