How to Agree to Disagree: Managing Ontological Perspectives using Standpoint Logic

Sebastian Rudolph, joint work with Lucía Gómez Álvarez, Hannes Strass, ...

International Center for Computational Logic

TECHNISCHE UNIVERSITÄT DRESDEN

Lucía Gómez Álvarez

"Queen of Standpoint Logic"
(inventor and major contributor to research on standpoint logic)

B. Bennett

S. Rudolph

J. Euzenat

PhD in Computer Science
University of Leeds, 2020
United Kingdom

Research Associate
TU Dresden (2020 - 2023)
Computational Logic Group
Germany

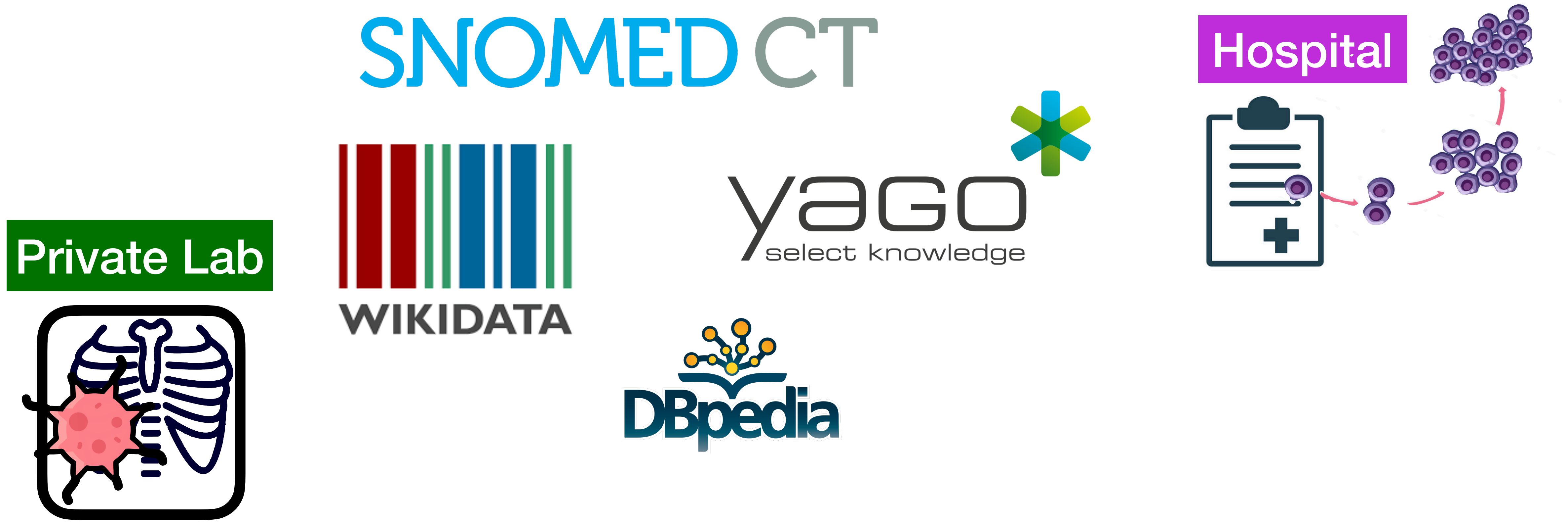
Chargée de Recherche
Université Grenoble Alpes (2024 - ...)
meOx team
France

Motivation

Multiperspective Reasoning

Motivation: Knowledge Integration

Motivation: Knowledge Integration



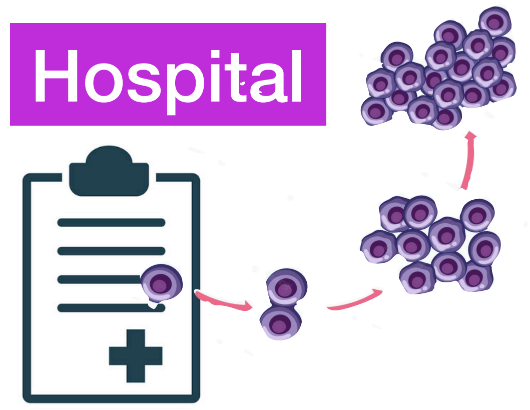
Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

SNOMED CT

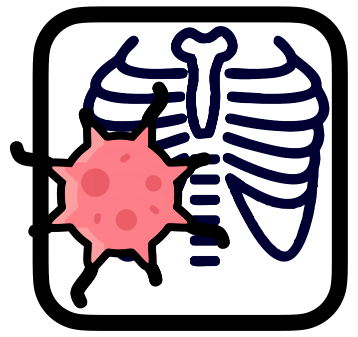
Hospital



WIKIDATA

yago
select knowledge

Private Lab



DBpedia

Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

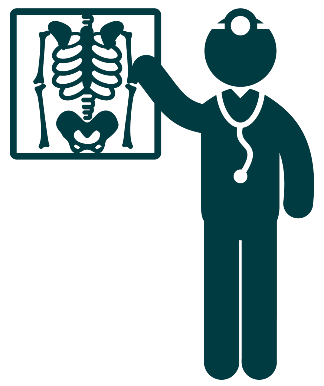
Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

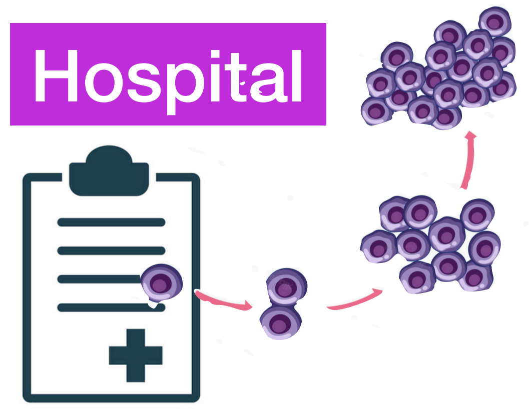
Private Lab

Tumor(t)
patientPart(p, t)

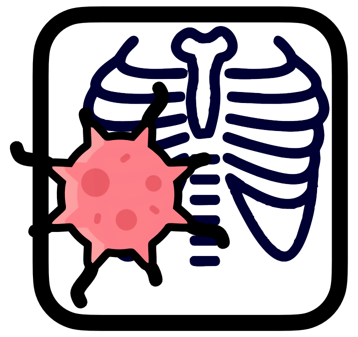


SNOMED CT

Hospital



Private Lab



HighRisk(p)

Hospital

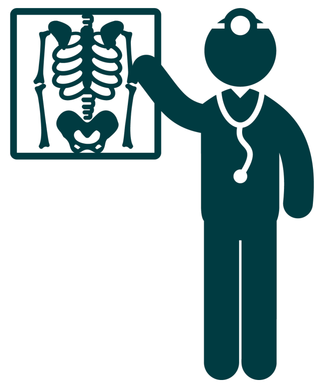
Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

Private Lab

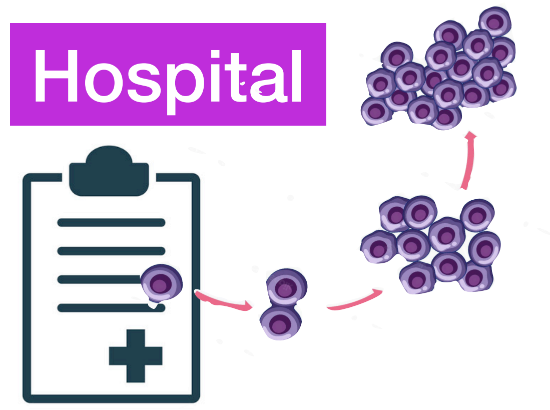
$\mathsf{Tumor}(t)$
$\mathsf{patientPart}(p, t)$



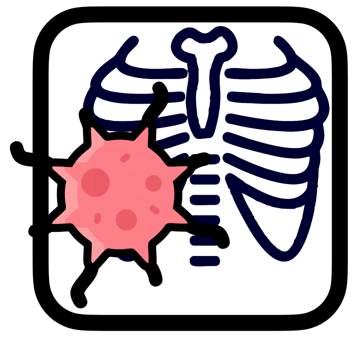
SNOMED CT

$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

Hospital



Private Lab



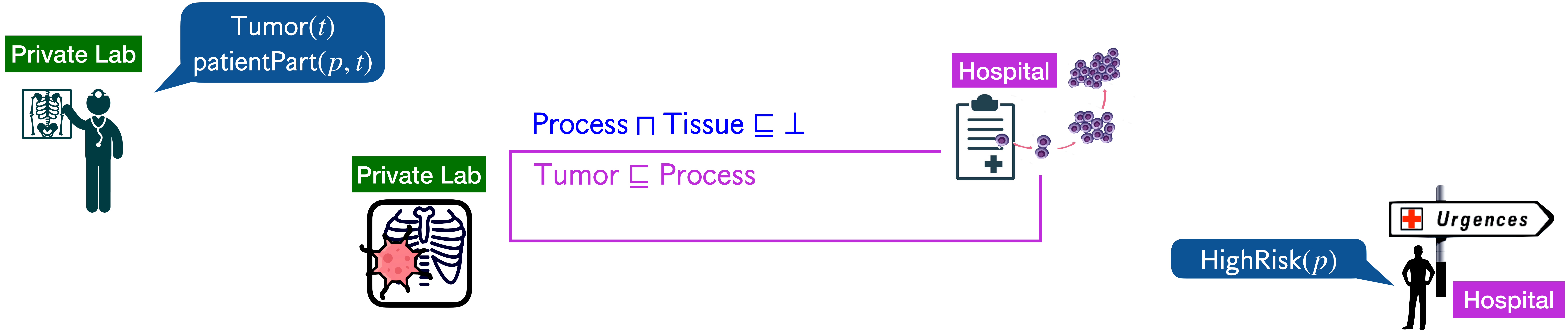
$\mathsf{HighRisk}(p)$

Hospital

Diverse Knowledge Sources

Motivation: Knowledge Integration

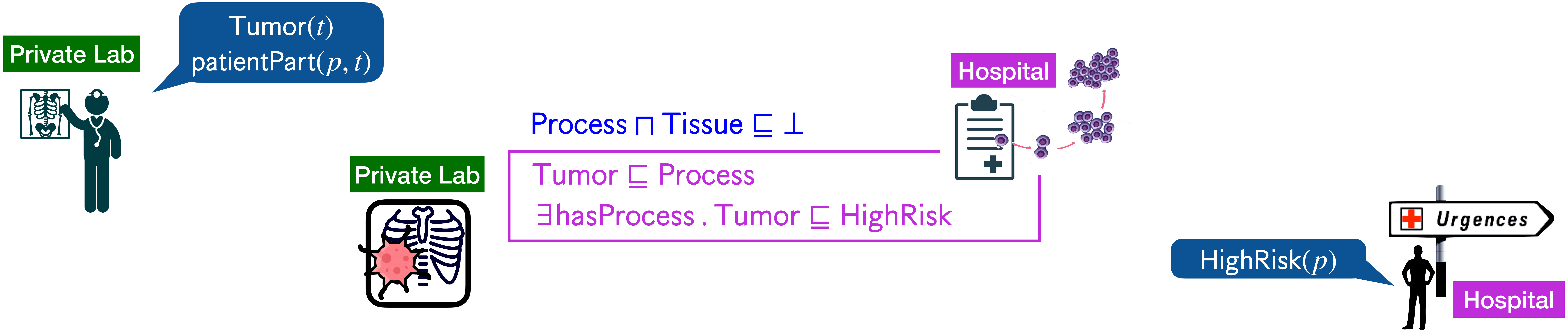
Non-trivial combinations of the huge diversity of knowledge sources available



Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available



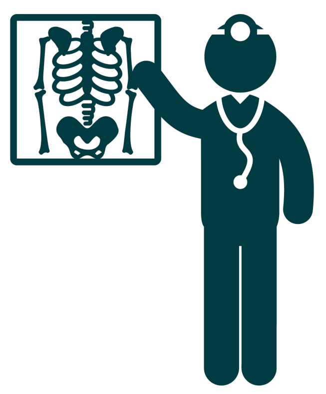
Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

Private Lab

$\mathsf{Tumor}(t)$
$\mathsf{patientPart}(p, t)$

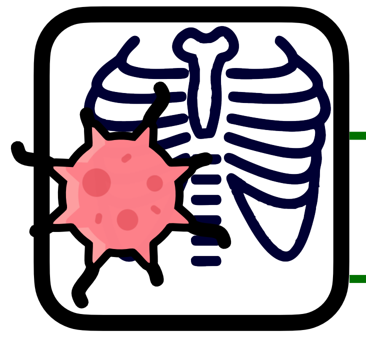


$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

Private Lab

$\mathsf{Tumor} \sqsubseteq \mathsf{Process}$

$\exists \mathsf{hasProcess}.\mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}$



$\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}$

$\mathsf{HighRisk}(p)$

Hospital

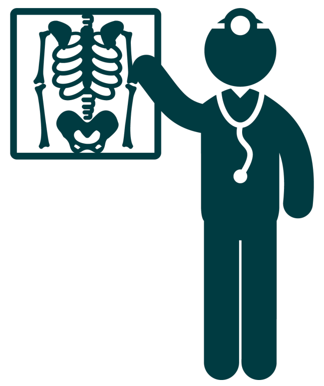
Diverse Knowledge Sources

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

Knowledge sources embed the perspectives of their creators!

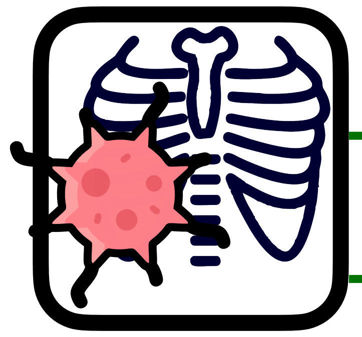
Private Lab



$\text{Tumor}(t)$
$\text{patientPart}(p, t)$

$\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot$

Private Lab



$\text{Tumor} \sqsubseteq \text{Process}$

$\exists \text{hasProcess} . \text{Tumor} \sqsubseteq \text{HighRisk}$

$\boxed{\text{Tumor} \sqsubseteq \text{Tissue}}$

$\text{HighRisk}(p)$

Hospital

Diverse Knowledge Sources

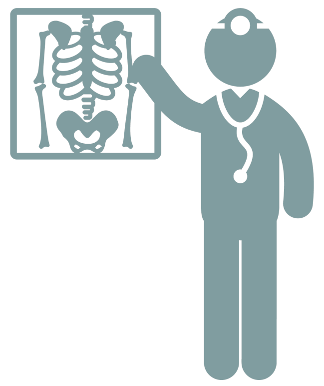
Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available
Knowledge sources embed the perspectives of their creators!

Challenge: Integration

Private Lab

$\mathsf{Tumor}(t)$
$\mathsf{patientPart}(p, t)$



$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

$\mathsf{Tumor} \sqsubseteq \mathsf{Process}$
$\exists \mathsf{hasProcess}.\mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}$

$\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}$

$\mathsf{HighRisk}(p)$

Hospital

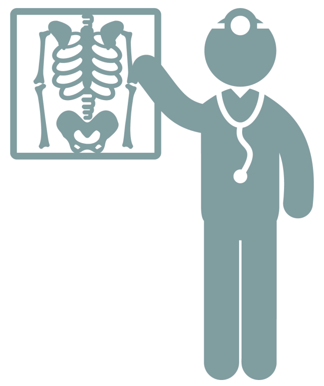
Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

Knowledge sources embed the perspectives of their creators!

Challenge: Integration

Private Lab



$\mathsf{Tumor}(t)$
$\mathsf{patientPart}(p,t)$

Knowledge Fusion

$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

$\mathsf{Tumor} \sqsubseteq \mathsf{Process}$
$\exists \mathsf{hasProcess}.\mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}$

$\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}$

$\mathsf{HighRisk}(p)$

Hospital

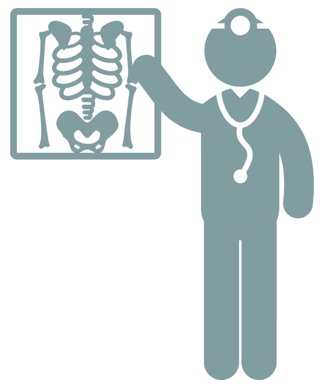
Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available
Knowledge sources embed the perspectives of their creators!

Challenge: Integration

$\text{Tumor}(t)$

Private Lab



$\text{Tumor}(t)$
$\text{patientPart}(p, t)$

Knowledge Fusion

$\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot$

$\text{Tumor} \sqsubseteq \text{Process}$

$\exists \text{hasProcess} . \text{Tumor} \sqsubseteq \text{HighRisk}$

$\text{Tumor} \sqsubseteq \text{Tissue}$

$\text{HighRisk}(p)$

Hospital

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

Knowledge sources embed the perspectives of their creators!

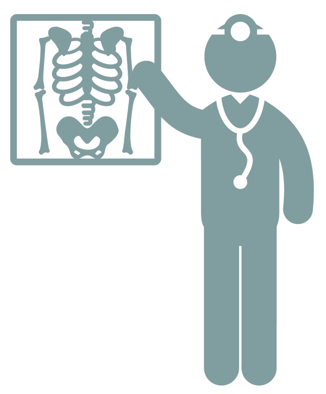
Challenge: Integration

$\mathsf{Tumor}(t)$

$\mathsf{Process}(t)$

Private Lab

$\mathsf{Tumor}(t)$
$\mathsf{patientPart}(p, t)$



Knowledge Fusion

$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

$\mathsf{Tumor} \sqsubseteq \mathsf{Process}$

$\exists \mathsf{hasProcess}.\mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}$

$\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}$

$\mathsf{HighRisk}(p)$

Hospital

Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available

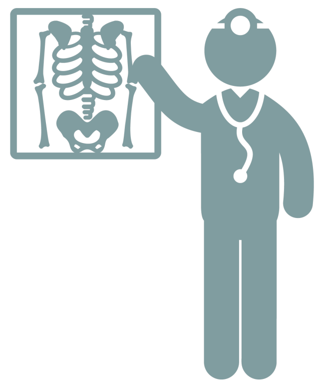
Knowledge sources embed the perspectives of their creators!

Challenge: Integration

Tumor(t)

Process(t) Tissue(t)

Private Lab



Tumor(t)
patientPart(p, t)

Knowledge Fusion

$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

$\mathsf{Tumor} \sqsubseteq \mathsf{Process}$

$\exists \mathsf{hasProcess} . \mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}$

$\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}$

HighRisk(p)

Hospital

Motivation: Knowledge Integration

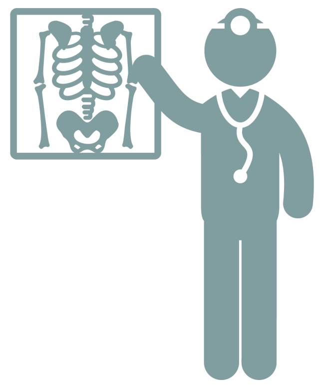
Non-trivial combinations of the huge diversity of knowledge sources available
Knowledge sources embed the perspectives of their creators!

Challenge: Integration

$$\begin{tikzcd} & \mathsf{Tumor}(t) \arrow[dl] \arrow[dr] & \\ \mathsf{Process}(t) \arrow[r] & \bot & \mathsf{Tissue}(t) \arrow[l] \end{tikzcd}$$

Private Lab

Tumor(t)
patientPart(p, t)



Knowledge Fusion

Process $\sqcap$ Tissue $\sqsubseteq \bot$

Tumor $\sqsubseteq$ Process

$\exists$hasProcess . Tumor $\sqsubseteq$ HighRisk

Tumor $\sqsubseteq$ Tissue

HighRisk(p)

Hospital

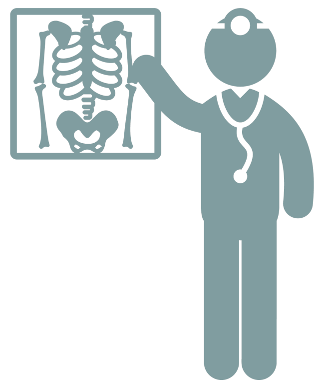
Motivation: Knowledge Integration

Non-trivial combinations of the huge diversity of knowledge sources available
Knowledge sources embed the perspectives of their creators!

Challenge: Integration

$$\begin{array}{ccccc} & & \mathsf{Tumor}(t) & & \\ & \swarrow & & \searrow & \\ \mathsf{Process}(t) & \longrightarrow & \bot & \longleftarrow & \mathsf{Tissue}(t) \end{array}$$

Private Lab



$\mathsf{Tumor}(t)$
$\mathsf{patientPart}(p, t)$

Knowledge Fusion

$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

$\mathsf{Tumor} \sqsubseteq \mathsf{Process}$
$\exists \mathsf{hasProcess} . \mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}$

$\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}$

$\mathsf{HighRisk}(p)$

Hospital

Multiperspective Ontology Management

Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

➡ **Multimodal logic** characterised by simplified Kripke semantics

Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

- ➡ **Multimodal logic** characterised by simplified Kripke semantics
- ➡ Knowledge relative to “points of view” (standpoints)

Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

- ➡ **Multimodal logic** characterised by simplified Kripke semantics
- ➡ Knowledge relative to "points of view" (standpoints)

$\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot$

$\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}$

$\mathsf{Tumor} \sqsubseteq \mathsf{Process}$

$\exists \mathsf{hasProcess} . \mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}$

Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

- ➡ **Multimodal logic** characterised by simplified Kripke semantics
- ➡ Knowledge relative to "points of view" (standpoints)

$\square_S [\mathrm{Process} \sqcap \mathrm{Tissue} \sqsubseteq \bot]$

$\lozenge_L [\mathrm{Tumor}] \sqsubseteq \square_L [\mathrm{Tissue}]$

$\lozenge_H [\mathrm{Tumor}] \sqsubseteq \square_H [\mathrm{Process}]$

$\square_H [\exists \mathrm{hasProcess}.\mathrm{Tumor} \sqsubseteq \mathrm{HighRisk}]$

$\square_e$ Unequivocal to e
$\lozenge_e$ Conceivable to e

Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

➡ **Multimodal logic** characterised by simplified Kripke semantics

➡ Knowledge relative to "points of view" (standpoints)

$\Box_S [\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot]$

$\Diamond_L [\text{Tumor}] \sqsubseteq \Box_L [\text{Tissue}]$

$\Diamond_H [\text{Tumor}] \sqsubseteq \Box_H [\text{Process}]$

$\Box_H [\exists \text{hasProcess} . \text{Tumor} \sqsubseteq \text{HighRisk}]$

$(L \cup H) \preceq S$

$\Box_e$ Unequivocal to e

$\Diamond_e$ Conceivable to e

(L and H inherit the axioms of S)

Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

- ➡ **Multimodal logic** characterised by simplified Kripke semantics
- ➡ Knowledge relative to "points of view" (standpoints)

CONSISTENT

$\square_e$ Unequivocal to e

$\lozenge_e$ Conceivable to e

$\square_S \left[\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot\right]$

$\lozenge_L \left[\text{Tumor}\right] \sqsubseteq \square_L \left[\text{Tissue}\right]$

$\lozenge_H \left[\text{Tumor}\right] \sqsubseteq \square_H \left[\text{Process}\right]$

$\square_H \left[\exists \text{hasProcess} . \text{Tumor} \sqsubseteq \text{HighRisk}\right]$

$(L \cup H) \preceq S$ (L and H inherit the axioms of S)

Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

- ➡ **Multimodal logic** characterised by simplified Kripke semantics
- ➡ Knowledge relative to "points of view" (standpoints)

CONSISTENT

$\Box_e$ Unequivocal to e

$\Diamond_e$ Conceivable to e

$\Box_S \left[\mathrm{Process} \sqcap \mathrm{Tissue} \sqsubseteq \bot\right]$

$\Diamond_L \left[\mathrm{Tumor}\right] \sqsubseteq \Box_L \left[\mathrm{Tissue}\right]$

$\Diamond_H \left[\mathrm{Tumor}\right] \sqsubseteq \Box_H \left[\mathrm{Process}\right]$

$\Box_H \left[\exists \mathrm{hasProcess} . \mathrm{Tumor} \sqsubseteq \mathrm{HighRisk}\right]$

$(L \cup H) \preceq S$ (L and H inherit the axioms of S)

$\Diamond_L \left[\exists \mathrm{patientPart} . \mathrm{Tumor}\right] \sqsubseteq \Box_H \left[\exists \mathrm{hasProcess} . \mathrm{Tumor}\right]$

Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

➡ **Multimodal logic** characterised by simplified Kripke semantics

➡ Knowledge relative to "points of view" (standpoints)

CONSISTENT

$\Box_e$ Unequivocal to e
$\Diamond_e$ Conceivable to e

$\Box_S\,[\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot\,]$

$\Diamond_L\,[\mathsf{Tumor}] \sqsubseteq \Box_L\,[\mathsf{Tissue}]$

$\Diamond_H\,[\mathsf{Tumor}] \sqsubseteq \Box_H\,[\mathsf{Process}]$

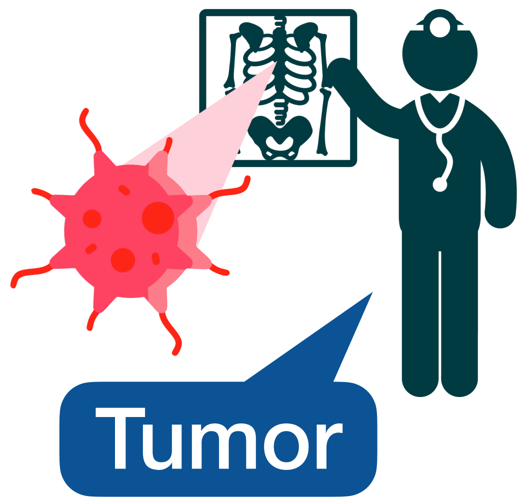
$\Box_H\,[\exists \mathsf{hasProcess}\,.\,\mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}]$

$(\,\mathrm{L} \cup \mathrm{H}\,) \preceq \mathrm{S}$ (L and H inherit the axioms of S)

$\Diamond_L\,[\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}] \sqsubseteq \Box_H\,[\exists \mathsf{hasProcess}\,.\,\mathsf{Tumor}]$

$\Diamond_L \mathsf{Tumor}(t)$ $\Box_L\,\mathsf{patientPart}(p, t)$

Private Lab



Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

➡ **Multimodal logic** characterised by simplified Kripke semantics

➡ Knowledge relative to “points of view” (standpoints)

CONSISTENT

$\Box_e$ Unequivocal to e
$\Diamond_e$ Conceivable to e

$$\Box_S [\mathrm{Process} \sqcap \mathrm{Tissue} \sqsubseteq \bot]$$

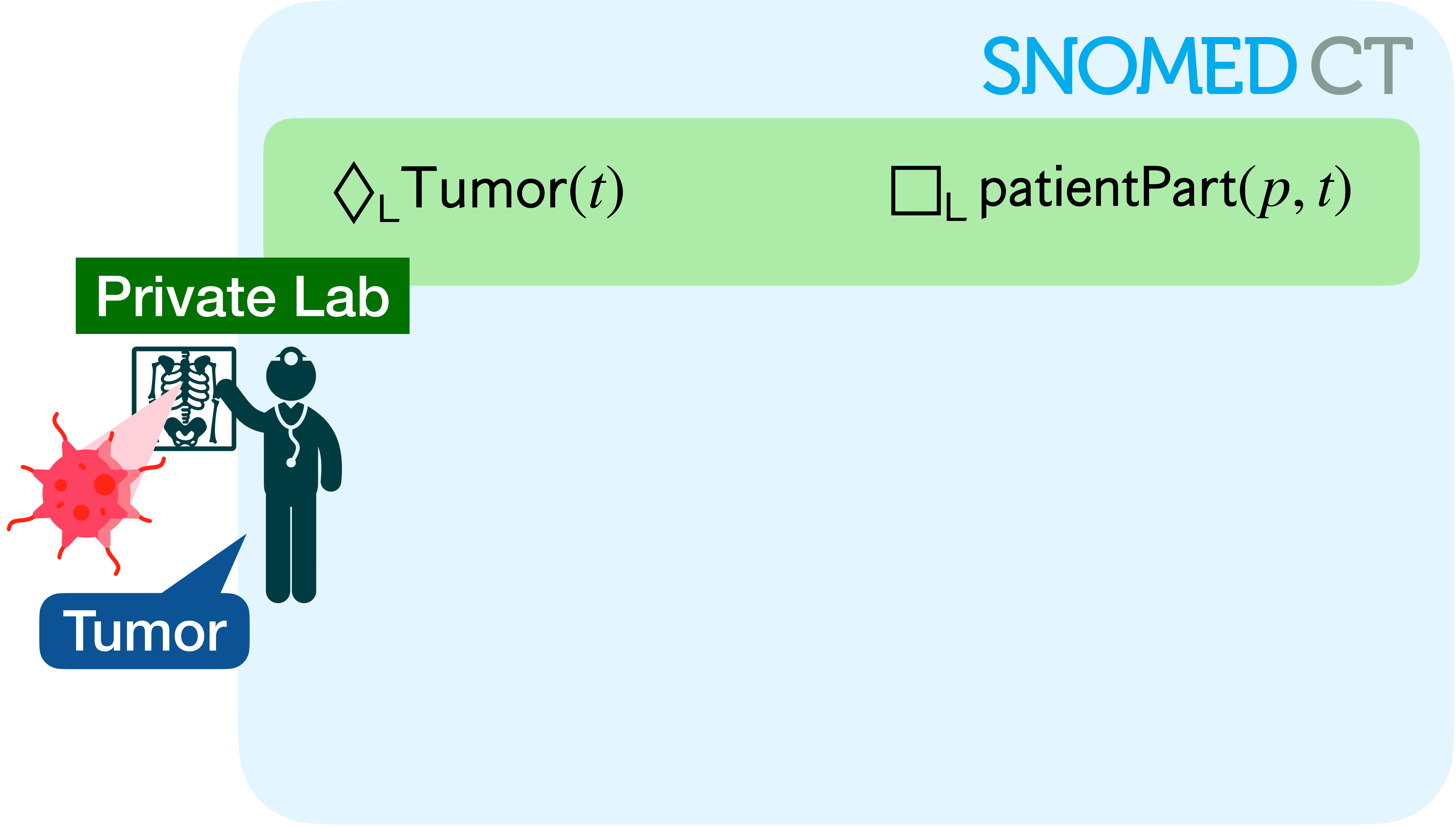
$$\Diamond_L [\mathrm{Tumor}] \sqsubseteq \Box_L [\mathrm{Tissue}]$$

$$\Diamond_H [\mathrm{Tumor}] \sqsubseteq \Box_H [\mathrm{Process}]$$

$$\Box_H [\exists \mathrm{hasProcess}\,.\,\mathrm{Tumor} \sqsubseteq \mathrm{HighRisk}]$$

$(L \cup H) \preceq S$ (L and H inherit the axioms of S)

$$\Diamond_L [\exists \mathrm{patientPart}\,.\,\mathrm{Tumor}] \sqsubseteq \Box_H [\exists \mathrm{hasProcess}\,.\,\mathrm{Tumor}]$$



Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

➡ **Multimodal logic** characterised by simplified Kripke semantics

➡ Knowledge relative to "points of view" (standpoints)

CONSISTENT

$\square_e$ Unequivocal to e

$\Diamond_e$ Conceivable to e

$\square_S [\text{Process} \sqcap \text{Tissue} \sqsubseteq \bot]$

$\Diamond_L [\text{Tumor}] \sqsubseteq \square_L [\text{Tissue}]$

$\Diamond_H [\text{Tumor}] \sqsubseteq \square_H [\text{Process}]$

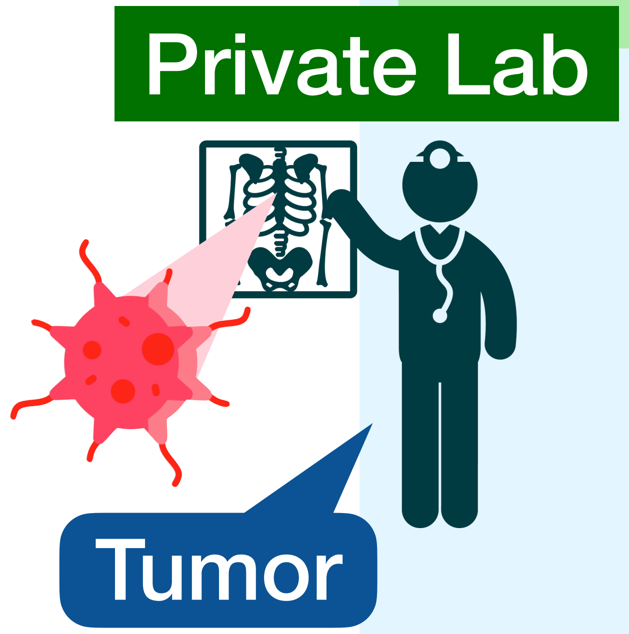
$\square_H [\exists \text{hasProcess} . \text{Tumor} \sqsubseteq \text{HighRisk}]$

$(L \cup H) \preceq S$ (L and H inherit the axioms of S)

$\Diamond_L [\exists \text{patientPart} . \text{Tumor}] \sqsubseteq \square_H [\exists \text{hasProcess} . \text{Tumor}]$

SNOMED CT

$\Diamond_L \text{Tumor}(t)$ $\square_L \text{patientPart}(p, t)$



Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

➡ **Multimodal logic** characterised by simplified Kripke semantics

➡ Knowledge relative to "points of view" (standpoints)

CONSISTENT

$\Box_e$ Unequivocal to e
$\Diamond_e$ Conceivable to e

$$\Box_S [\mathrm{Process} \sqcap \mathrm{Tissue} \sqsubseteq \bot]$$

$$\Diamond_L [\mathrm{Tumor}] \sqsubseteq \Box_L [\mathrm{Tissue}]$$

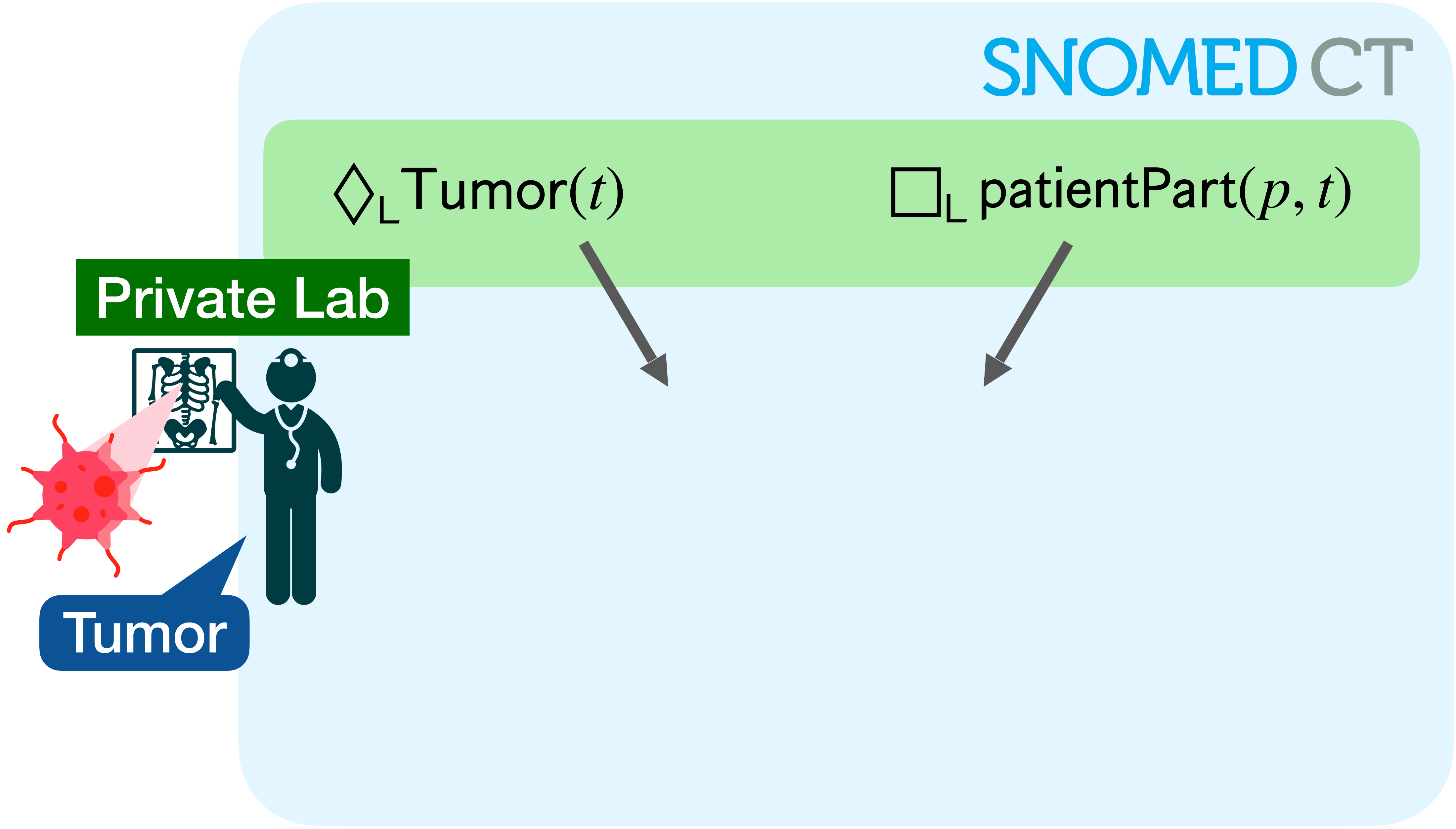
$$\Diamond_H [\mathrm{Tumor}] \sqsubseteq \Box_H [\mathrm{Process}]$$

$$\Box_H [\exists \mathrm{hasProcess} . \mathrm{Tumor} \sqsubseteq \mathrm{HighRisk}]$$

$$(L \cup H) \preceq S$$

(L and H inherit the axioms of S)

$$\Diamond_L [\exists \mathrm{patientPart} . \mathrm{Tumor}] \sqsubseteq \Box_H [\exists \mathrm{hasProcess} . \mathrm{Tumor}]$$



Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

➡ **Multimodal logic** characterised by simplified Kripke semantics

➡ Knowledge relative to “points of view” (standpoints)

$\Box_S\,[\mathrm{Process} \sqcap \mathrm{Tissue} \sqsubseteq \bot]$

$\Diamond_L\,[\mathrm{Tumor}] \sqsubseteq \Box_L\,[\mathrm{Tissue}]$

$\Diamond_H\,[\mathrm{Tumor}] \sqsubseteq \Box_H\,[\mathrm{Process}]$

$\Box_H\,[\exists \mathrm{hasProcess}\,.\,\mathrm{Tumor} \sqsubseteq \mathrm{HighRisk}]$

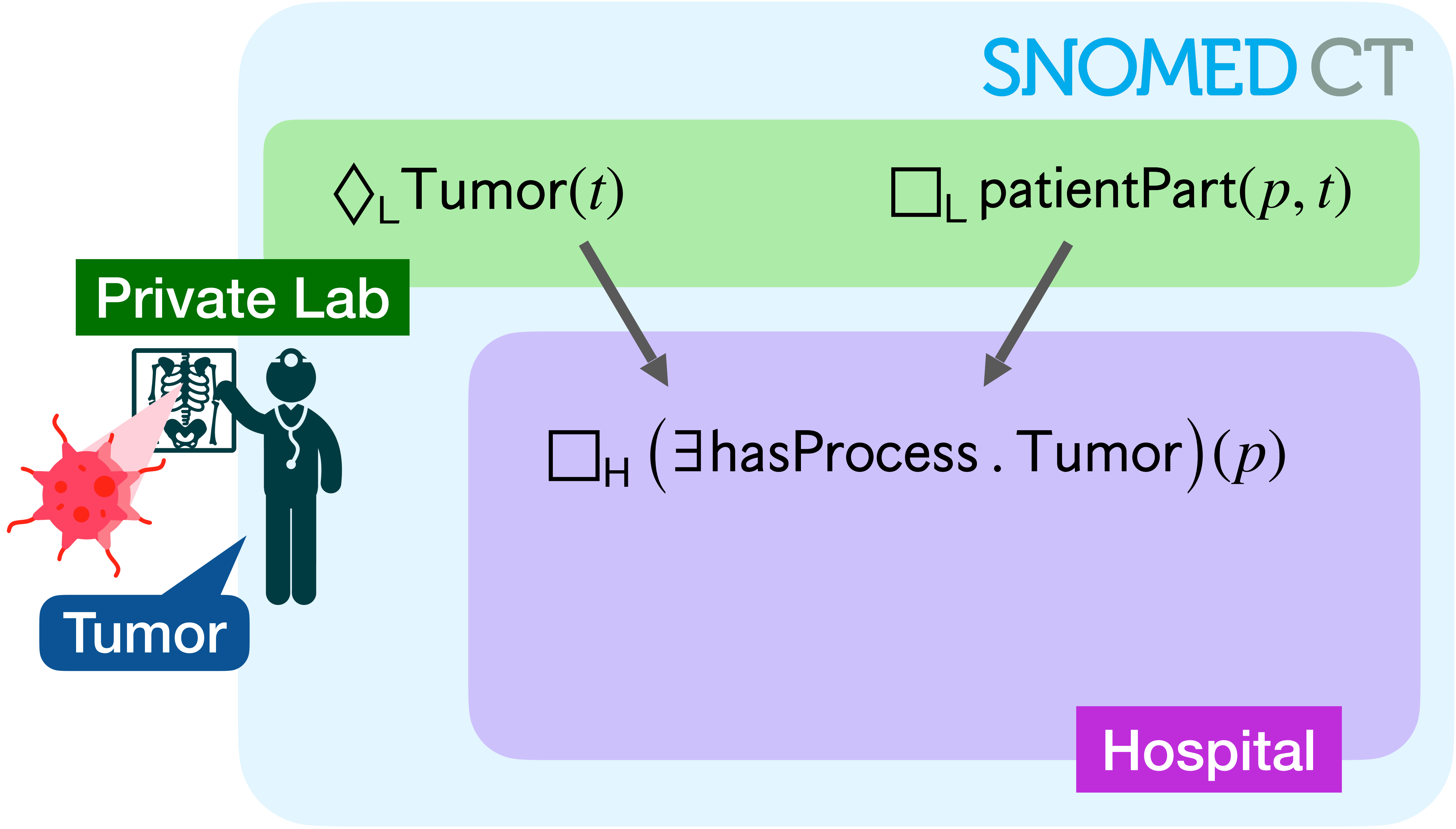
$(L \cup H) \preceq S$ (L and H inherit the axioms of S)

$\Diamond_L\,[\exists \mathrm{patientPart}\,.\,\mathrm{Tumor}] \sqsubseteq \Box_H\,[\exists \mathrm{hasProcess}\,.\,\mathrm{Tumor}]$

CONSISTENT

$\Box_e$ Unequivocal to e

$\Diamond_e$ Conceivable to e



Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

- ➡ **Multimodal logic** characterised by simplified Kripke semantics
- ➡ Knowledge relative to "points of view" (standpoints)

CONSISTENT

$\Box_e$ Unequivocal to e
$\Diamond_e$ Conceivable to e

$$\Box_S [\mathrm{Process} \sqcap \mathrm{Tissue} \sqsubseteq \bot]$$

$$\Diamond_L [\mathrm{Tumor}] \sqsubseteq \Box_L [\mathrm{Tissue}]$$

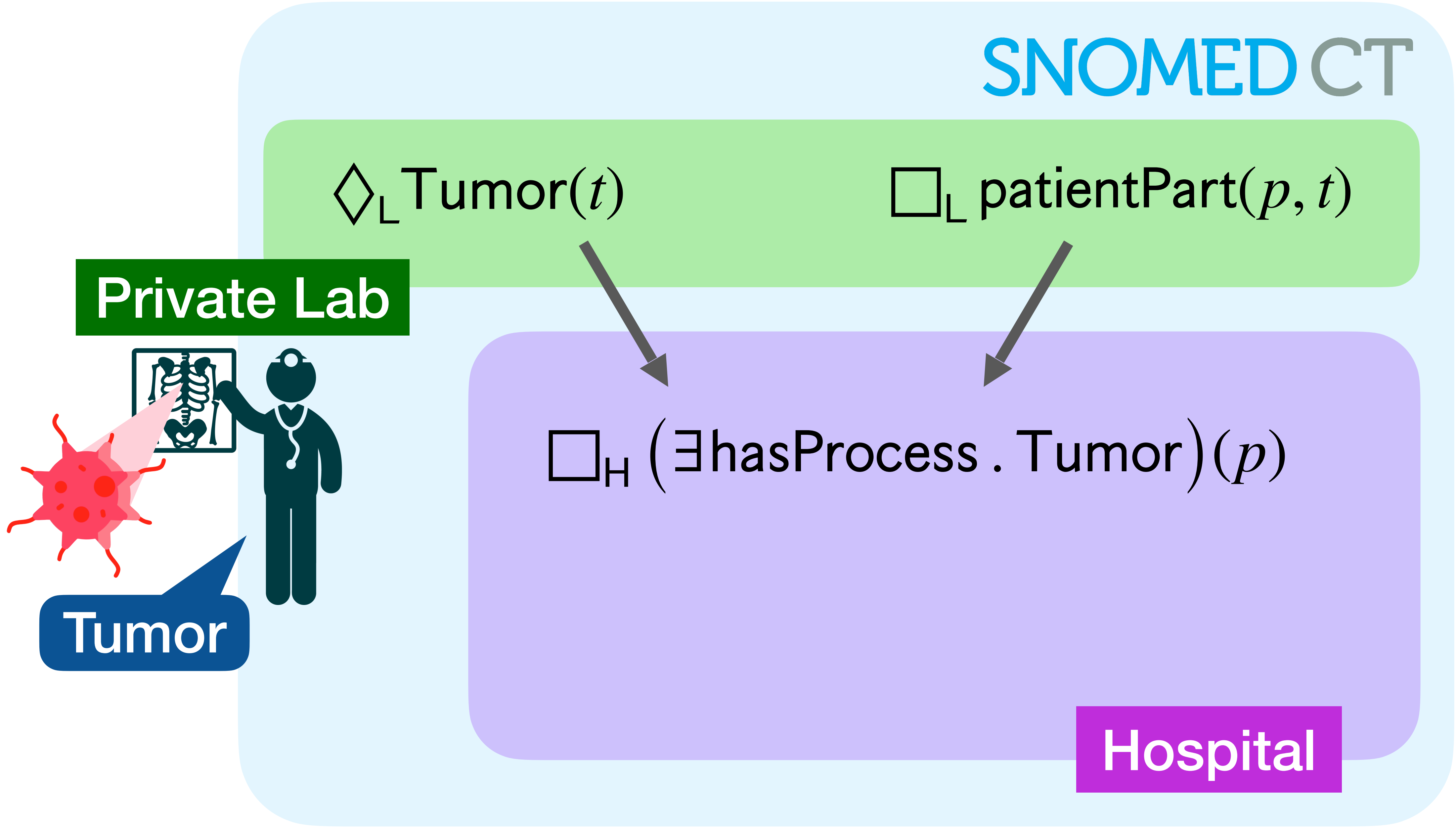
$$\Diamond_H [\mathrm{Tumor}] \sqsubseteq \Box_H [\mathrm{Process}]$$

$$\Box_H [\exists \mathrm{hasProcess} . \mathrm{Tumor} \sqsubseteq \mathrm{HighRisk}]$$

$$(L \cup H) \preceq S$$

(L and H inherit the axioms of S)

$$\Diamond_L [\exists \mathrm{patientPart} . \mathrm{Tumor}] \sqsubseteq \Box_H [\exists \mathrm{hasProcess} . \mathrm{Tumor}]$$



Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

➡ **Multimodal logic** characterised by simplified Kripke semantics

➡ Knowledge relative to "points of view" (standpoints)

$\Box_S\,[\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot]$

$\Diamond_L\,[\mathsf{Tumor}] \sqsubseteq \Box_L\,[\mathsf{Tissue}]$

$\Diamond_H\,[\mathsf{Tumor}] \sqsubseteq \Box_H\,[\mathsf{Process}]$

$\Box_H\,[\exists \mathsf{hasProcess}\,.\,\mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}]$

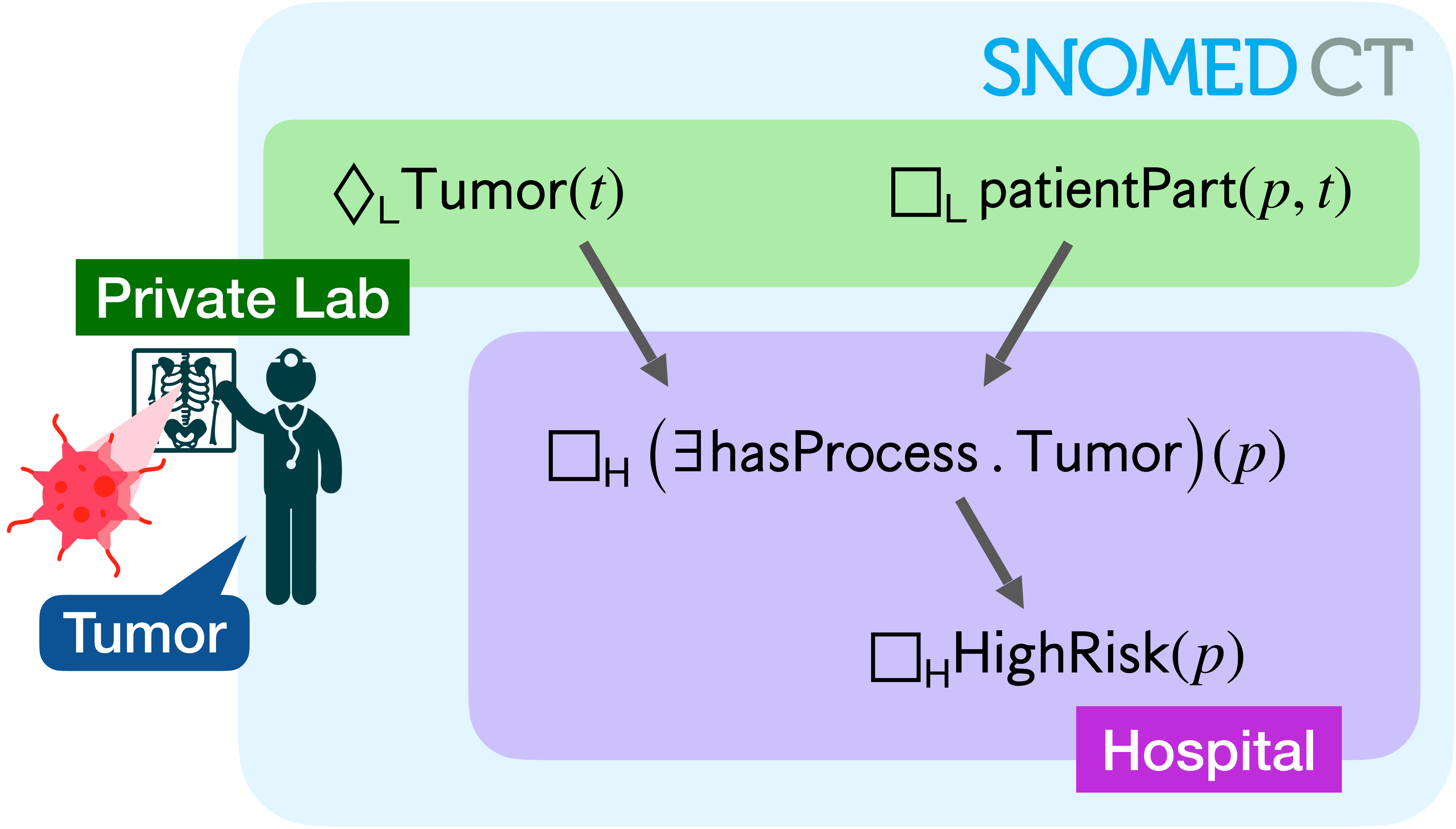
$(L \cup H) \preceq S$ (L and H inherit the axioms of S)

$\Diamond_L\,[\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}] \sqsubseteq \Box_H\,[\exists \mathsf{hasProcess}\,.\,\mathsf{Tumor}]$

CONSISTENT

$\Box_e$ Unequivocal to e

$\Diamond_e$ Conceivable to e



Multiperspective Ontology Management

Challenge: combining diverse (potentially conflicting) sources without weakening them

Standpoint Logic

➡ **Multimodal logic** characterised by simplified Kripke semantics

➡ Knowledge relative to "points of view" (standpoints)

CONSISTENT

$\Box_e$ Unequivocal to e

$\Diamond_e$ Conceivable to e

$\Box_S\,[\mathsf{Process} \sqcap \mathsf{Tissue} \sqsubseteq \bot]$

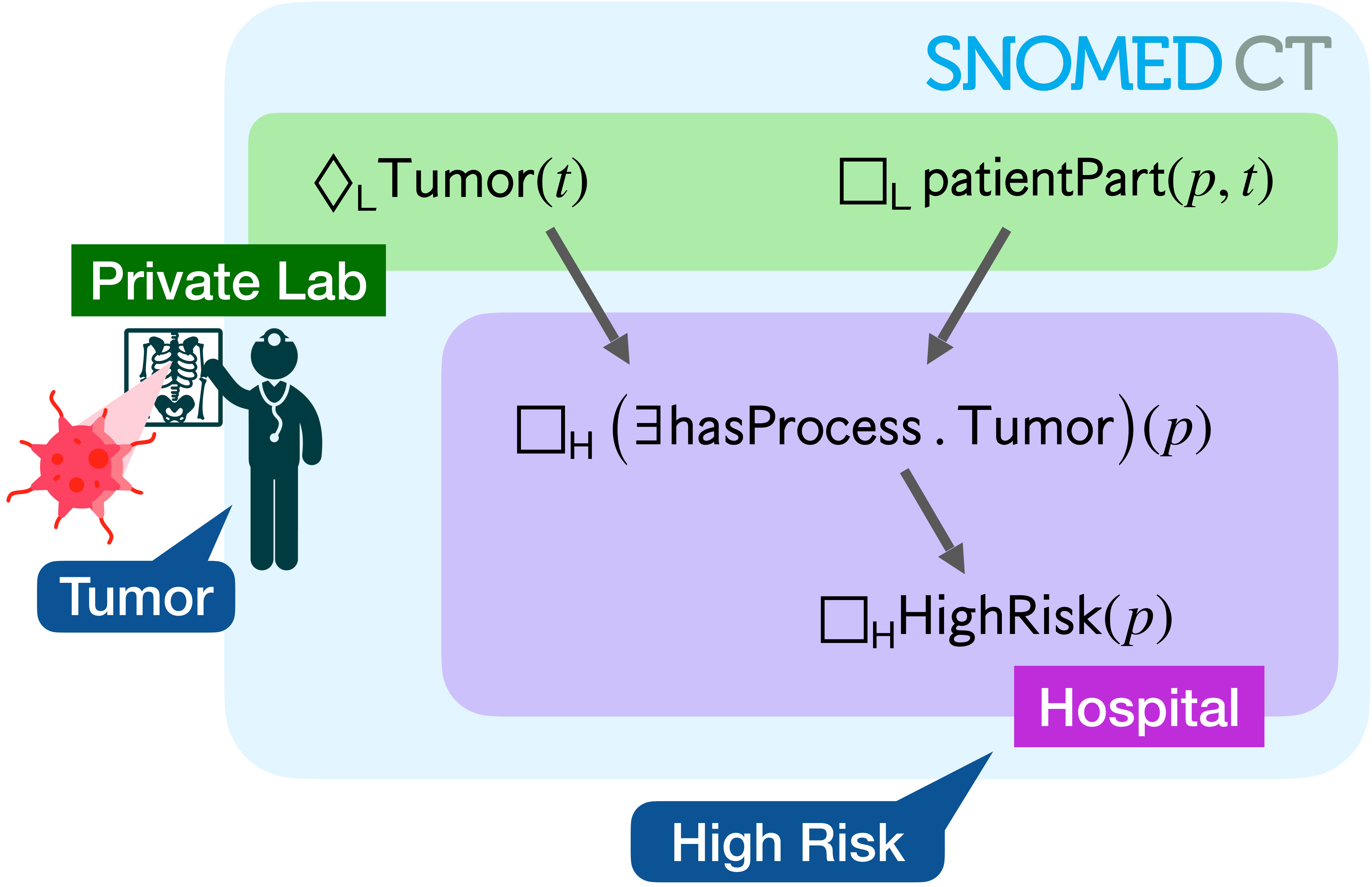
$\Diamond_L\,[\mathsf{Tumor}] \sqsubseteq \Box_L\,[\mathsf{Tissue}]$

$\Diamond_H\,[\mathsf{Tumor}] \sqsubseteq \Box_H\,[\mathsf{Process}]$

$\Box_H\,[\exists \mathsf{hasProcess}\,.\,\mathsf{Tumor} \sqsubseteq \mathsf{HighRisk}]$

$(\mathsf{L} \cup \mathsf{H}) \preceq \mathsf{S}$ (L and H inherit the axioms of S)

$\Diamond_L\,[\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}] \sqsubseteq \Box_H\,[\exists \mathsf{hasProcess}\,.\,\mathsf{Tumor}]$



Standpoint FOL – Syntax and Semantics

First-Order Standpoint Logic: Syntax

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

- $\Box_e \phi \longrightarrow$ "it is **unequivocal**, according to *e*, that ϕ"

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \land \psi \mid \forall x \phi \mid \square_e \phi$$

$$\square_e \neg\phi \equiv \neg\lozenge_e \phi \text{ (dual)}$$

- $\square_e \phi \longrightarrow$ "it is **unequivocal**, according to e, that ϕ"
- $\lozenge_e \phi \longrightarrow$ "it is **conceivable**, according to e, that ϕ"

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \square_e \phi$$

$$\square_e \neg\phi \equiv \neg\lozenge_e \phi \text{ (dual)}$$

The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

- $\square_e \phi \longrightarrow$ "it is **unequivocal**, according to e, that ϕ"
- $\lozenge_e \phi \longrightarrow$ "it is **conceivable**, according to e, that ϕ"

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \square_e \phi$$

$$\square_e \neg\phi \equiv \neg\lozenge_e \phi \text{ (dual)}$$

The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

- $\square_e \phi \longrightarrow$ "it is **unequivocal**, according to e, that ϕ"
- $\lozenge_e \phi \longrightarrow$ "it is **conceivable**, according to e, that ϕ"
- $\square_{e \cup e'} \phi \longrightarrow$ "it is unequivocal, <u>according to both e and e'</u>, that ϕ"

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

$$\Box_e \neg\phi \equiv \neg\Diamond_e \phi \text{ (dual)}$$

The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

- $\Box_e \phi \longrightarrow$ "it is **unequivocal**, according to e, that ϕ"
- $\Diamond_e \phi \longrightarrow$ "it is **conceivable**, according to e, that ϕ"
- $\Box_{e \cup e'} \phi \longrightarrow$ "it is unequivocal, according to both e and e', that ϕ"
- $\Box_{e \cap e'} \phi \longrightarrow$ "it is unequivocal, according to the fusion of e and e', that ϕ"

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

$$\Box_e \neg\phi \equiv \neg\Diamond_e \phi \text{ (dual)}$$

The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

+ (definable) sharpening statements: $e \preceq e'$ denoting $\Box_{e \backslash e'} \mathbf{f}$

- $\Box_e \phi \longrightarrow$ "it is **unequivocal**, according to e, that ϕ"
- $\Diamond_e \phi \longrightarrow$ "it is **conceivable**, according to e, that ϕ"
- $\Box_{e \cup e'} \phi \longrightarrow$ "it is unequivocal, <u>according to both e and e'</u>, that ϕ"
- $\Box_{e \cap e'} \phi \longrightarrow$ "it is unequivocal, <u>according to the fusion of e and e'</u>, that ϕ"

First-Order Standpoint Logic: Syntax

Syntax of $\mathbb{S}$

Signature $\langle \mathscr{P}, \mathscr{C}, \mathscr{S} \rangle$ of predicates, constants and standpoints.

The set of FOSL formulas is then given by

$$\phi, \psi ::= P(t_1, \ldots, t_k) \mid \neg\phi \mid \phi \wedge \psi \mid \forall x \phi \mid \Box_e \phi$$

$$\Box_e \neg\phi \equiv \neg\Diamond_e \phi \text{ (dual)}$$

The set of standpoint expressions is defined as follows:

$$e_1, e_2 ::= * \mid s \mid e_1 \cup e_2 \mid e_1 \cap e_2 \mid e_1 \backslash e_2$$

+ (definable) sharpening statements: $e \preceq e'$ denoting $\Box_{e \backslash e'} \mathbf{f}$

- $\Box_e \phi \longrightarrow$ "it is **unequivocal**, according to e, that ϕ"
- $\Diamond_e \phi \longrightarrow$ "it is **conceivable**, according to e, that ϕ"
- $\Box_{e \cup e'} \phi \longrightarrow$ "it is unequivocal, <u>according to both e and e'</u>, that ϕ"
- $\Box_{e \cap e'} \phi \longrightarrow$ "it is unequivocal, <u>according to the fusion of e and e'</u>, that ϕ"
- $e \preceq e' \longrightarrow$ "*e* inherits or **extends** *e'*"

First-Order Standpoint Logic: Semantics

First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathcal{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$

First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathcal{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$

First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$

ϵ_1	ϵ_2	Δ / Π
		π_1
		π_2
		π_3

First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathcal{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\text{R}) = \{\pi_1, \pi_2\}$

ϵ_1	ϵ_2	Δ / Π
		π_1
		π_2
		π_3

First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathrm{R}) = \{\pi_1, \pi_2\}$

ϵ_1	ϵ_2	Δ / Π	
		π_1	R
		π_2	
		π_3	

First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\text{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\text{B}) = \{\pi_2, \pi_3\}$

ϵ_1	ϵ_2	Δ / Π	
		π_1	R
		π_2	
		π_3	

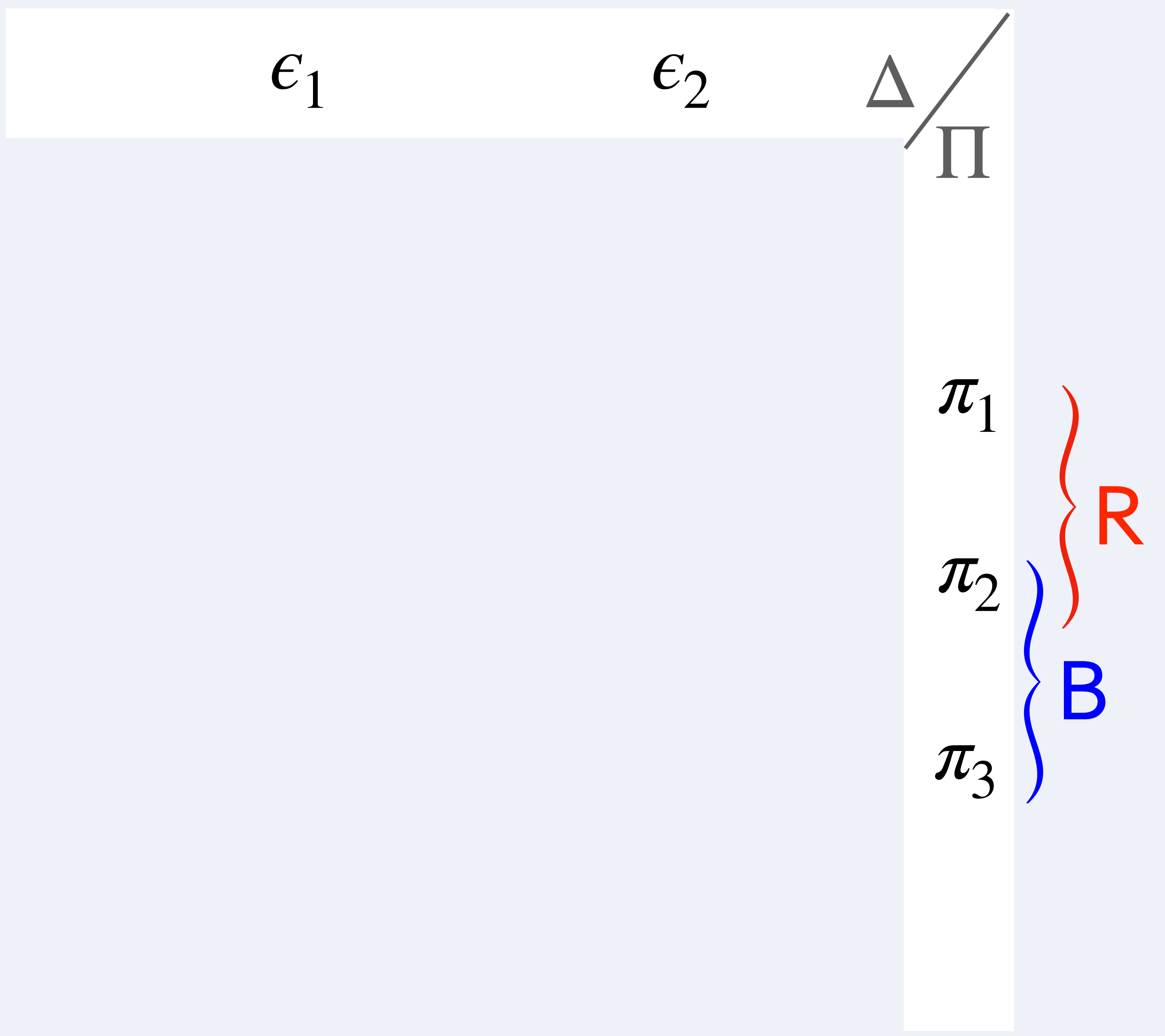
First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathcal{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\text{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\text{B}) = \{\pi_2, \pi_3\}$



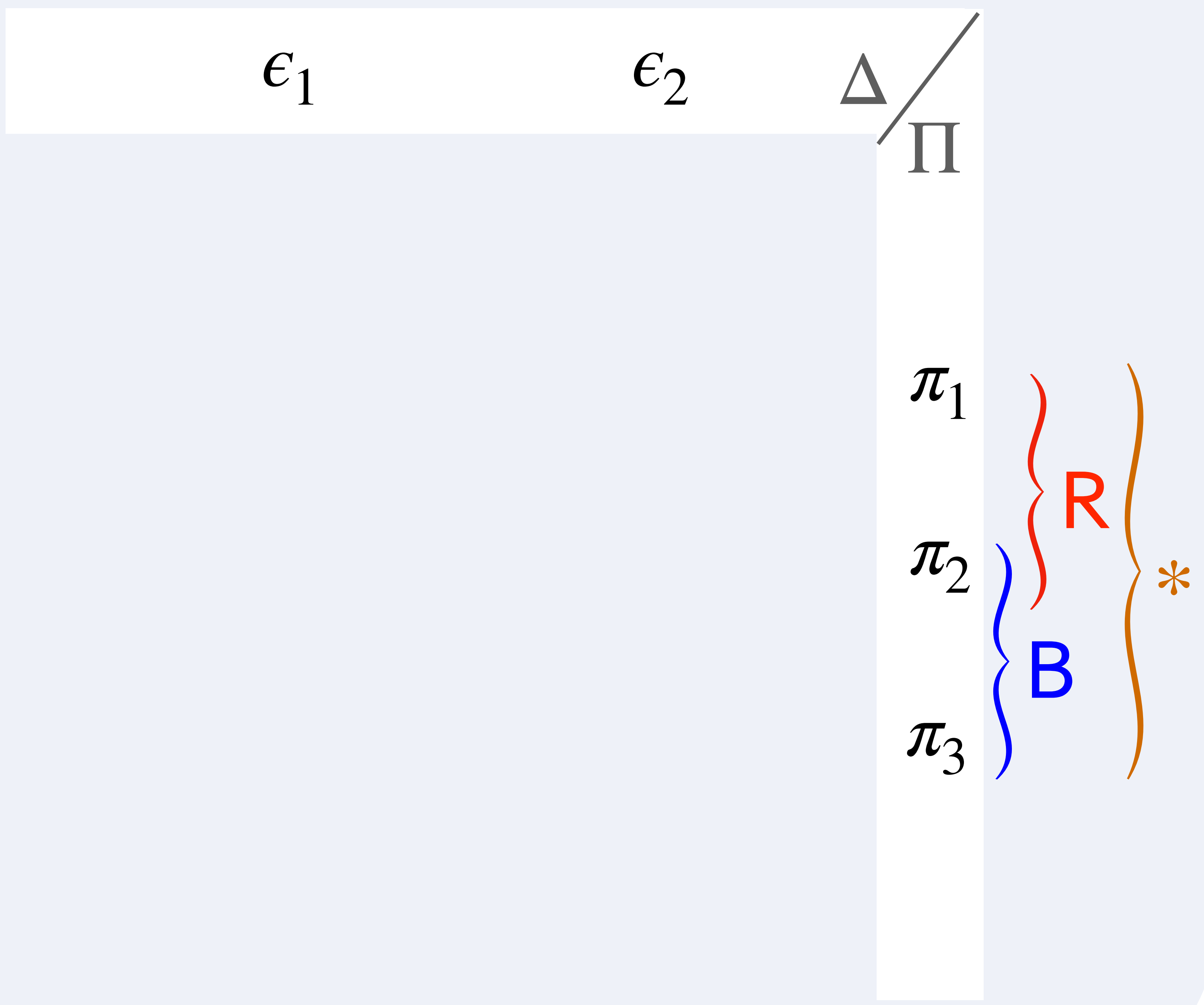
First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathsf{B}) = \{\pi_2, \pi_3\}$



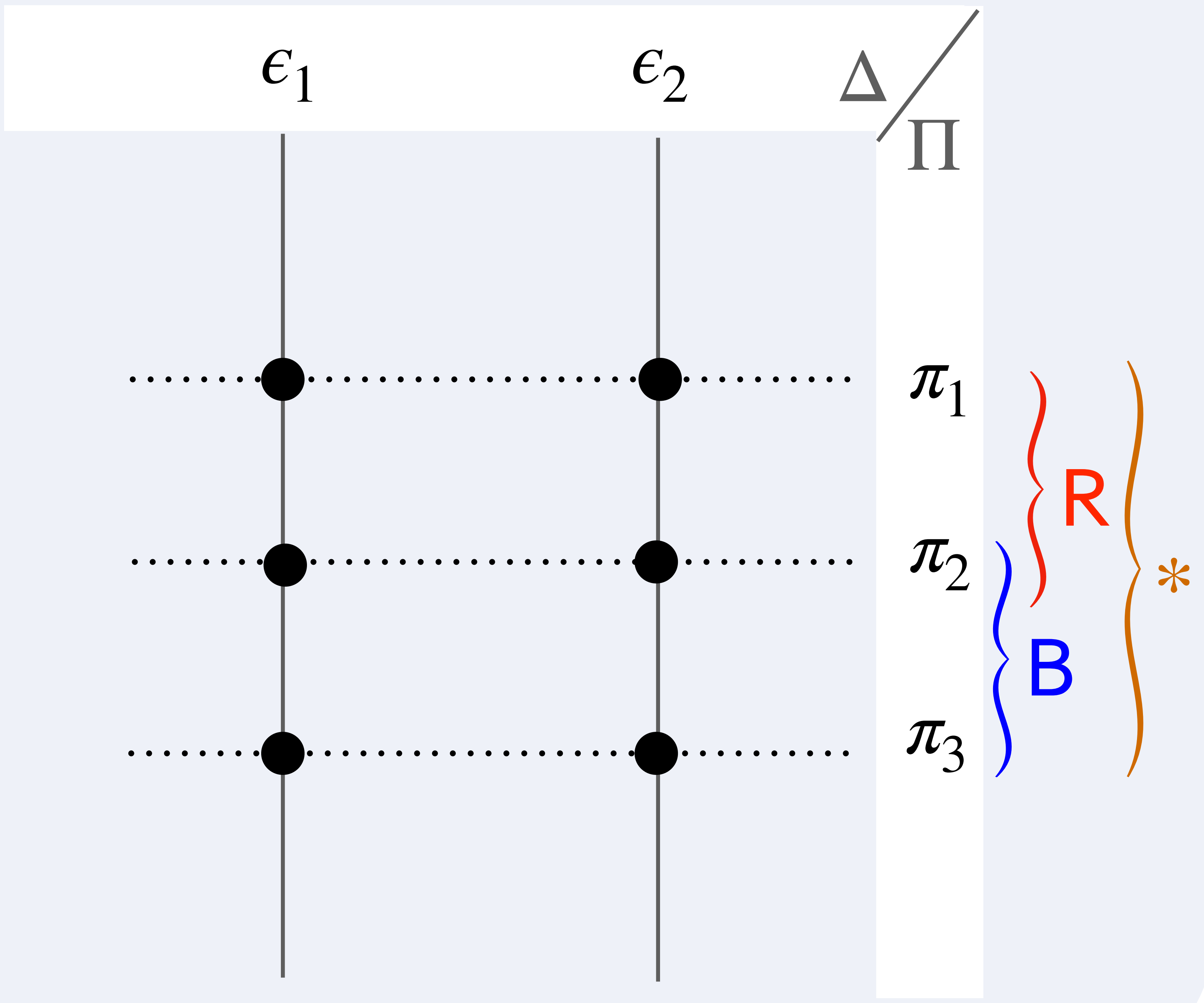
First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

Relational semantics:

$$\mathcal{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathsf{B}) = \{\pi_2, \pi_3\}$
- $\gamma(\pi_1) = (p \mapsto \emptyset, \ldots)$



First-Order Standpoint Logic: Semantics

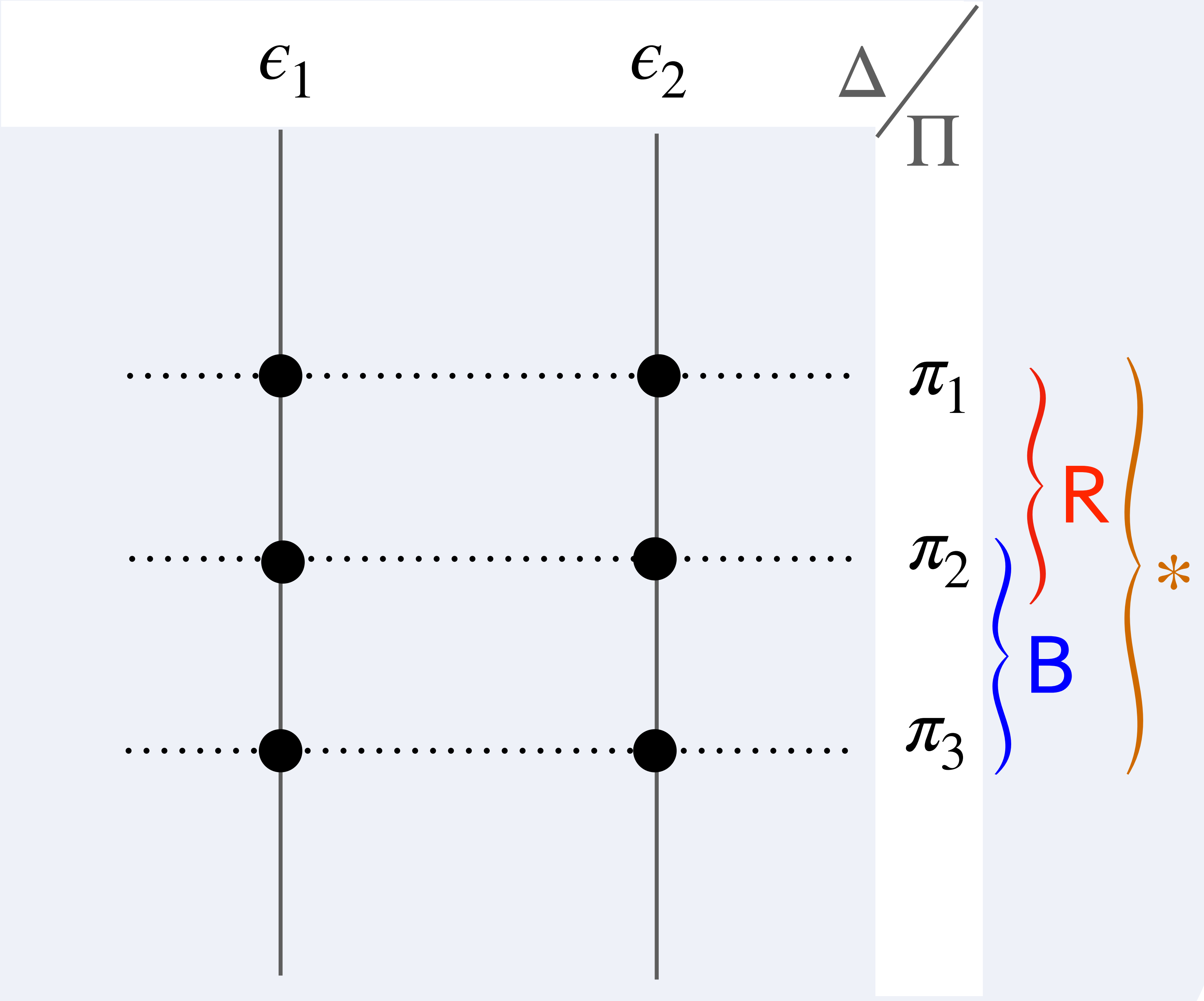
Semantics of $\mathbb{S}$

Relational semantics:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathsf{B}) = \{\pi_2, \pi_3\}$
- $\gamma(\pi_1) = (p \mapsto \varnothing, \dots)$

*Rigid domains and constants



First-Order Standpoint Logic: Semantics

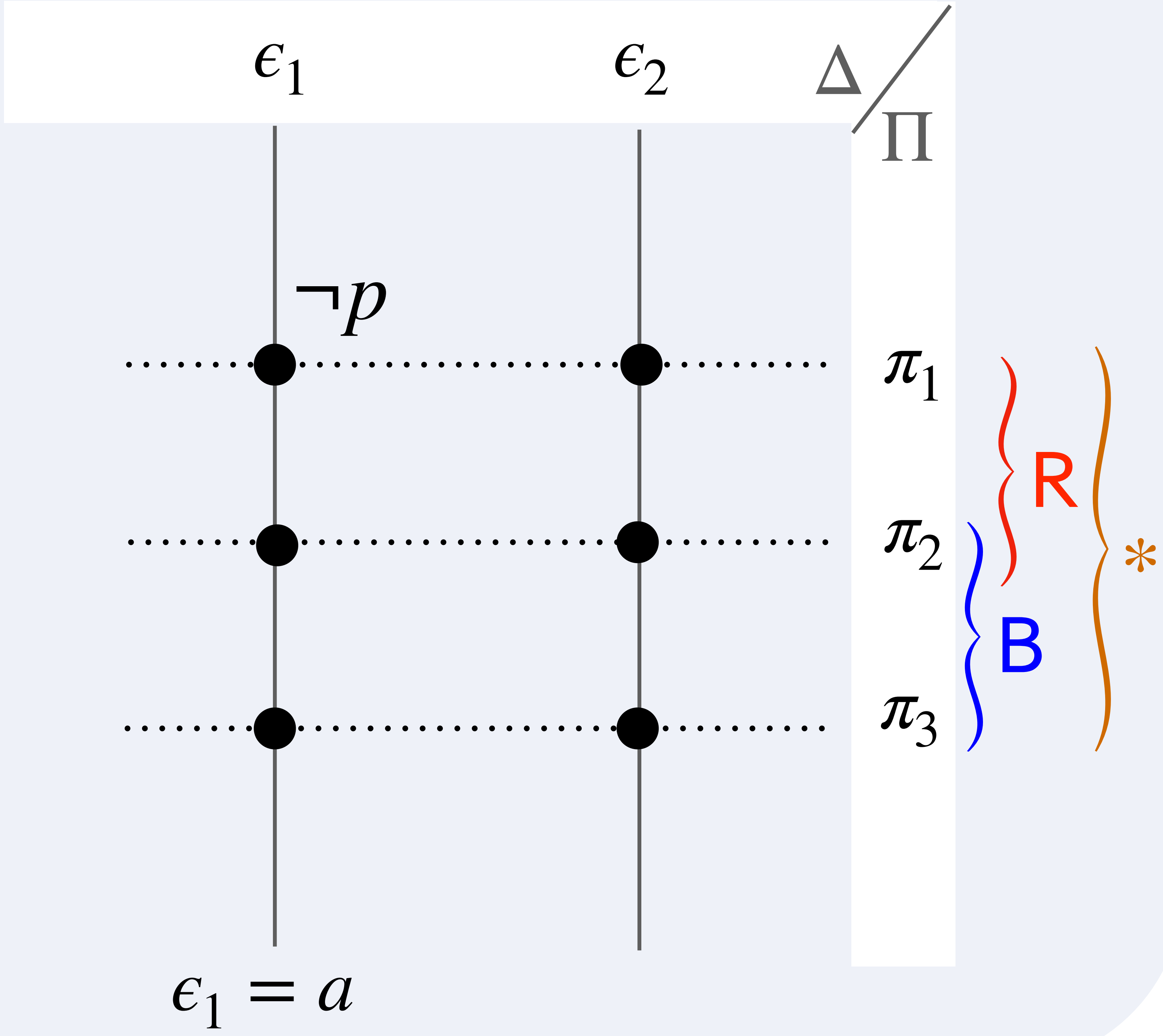
Semantics of $\mathbb{S}$

Relational semantics:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathsf{B}) = \{\pi_2, \pi_3\}$
- $\gamma(\pi_1) = (p \mapsto \varnothing, \dots)$

*Rigid domains and constants



First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

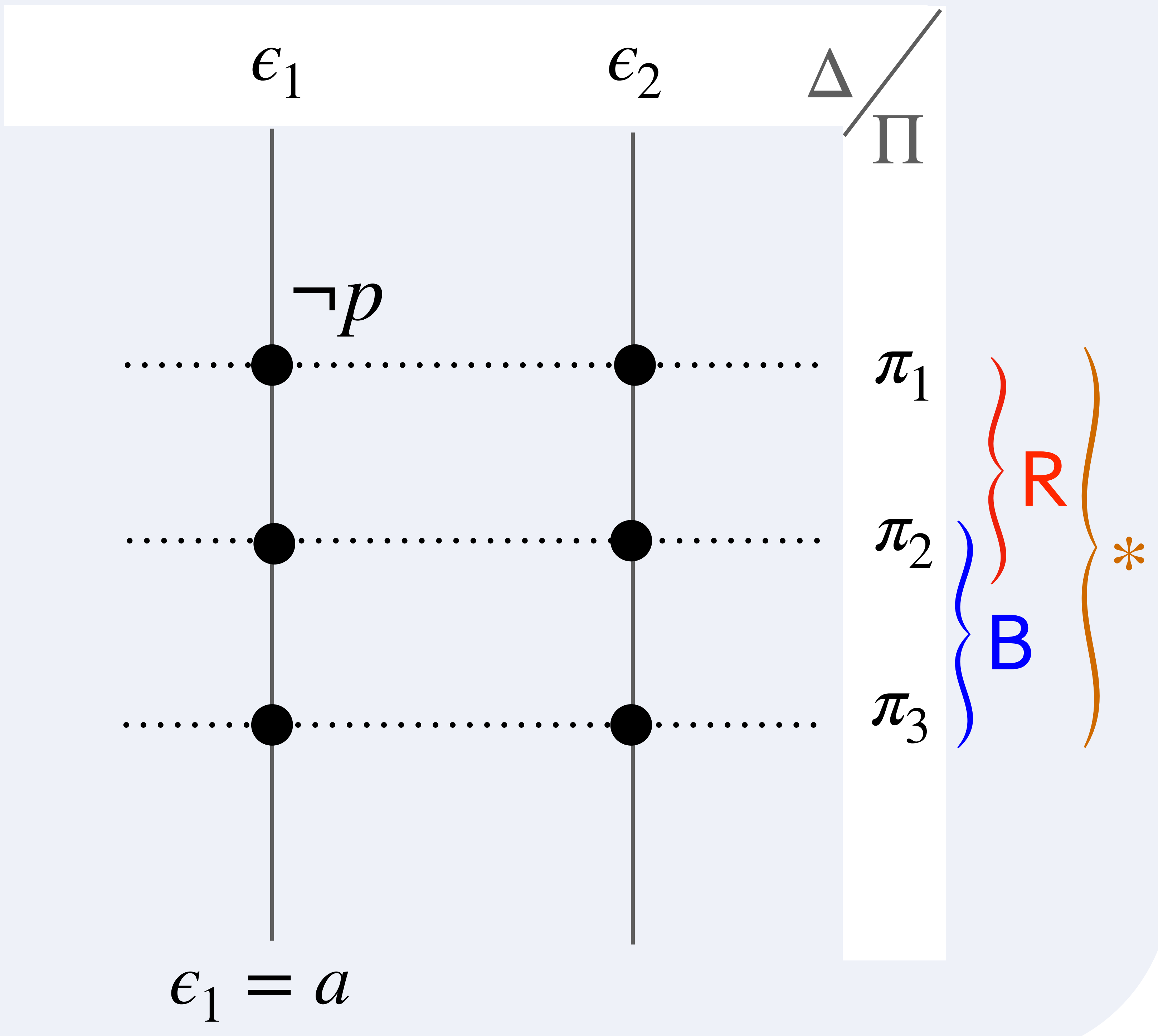
Relational semantics:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathsf{B}) = \{\pi_2, \pi_3\}$
- $\gamma(\pi_1) = (p \mapsto \emptyset, \dots)$

- $\mathscr{M} \models \Box_{\mathsf{B}}\, p(a)$

*Rigid domains and constants



First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

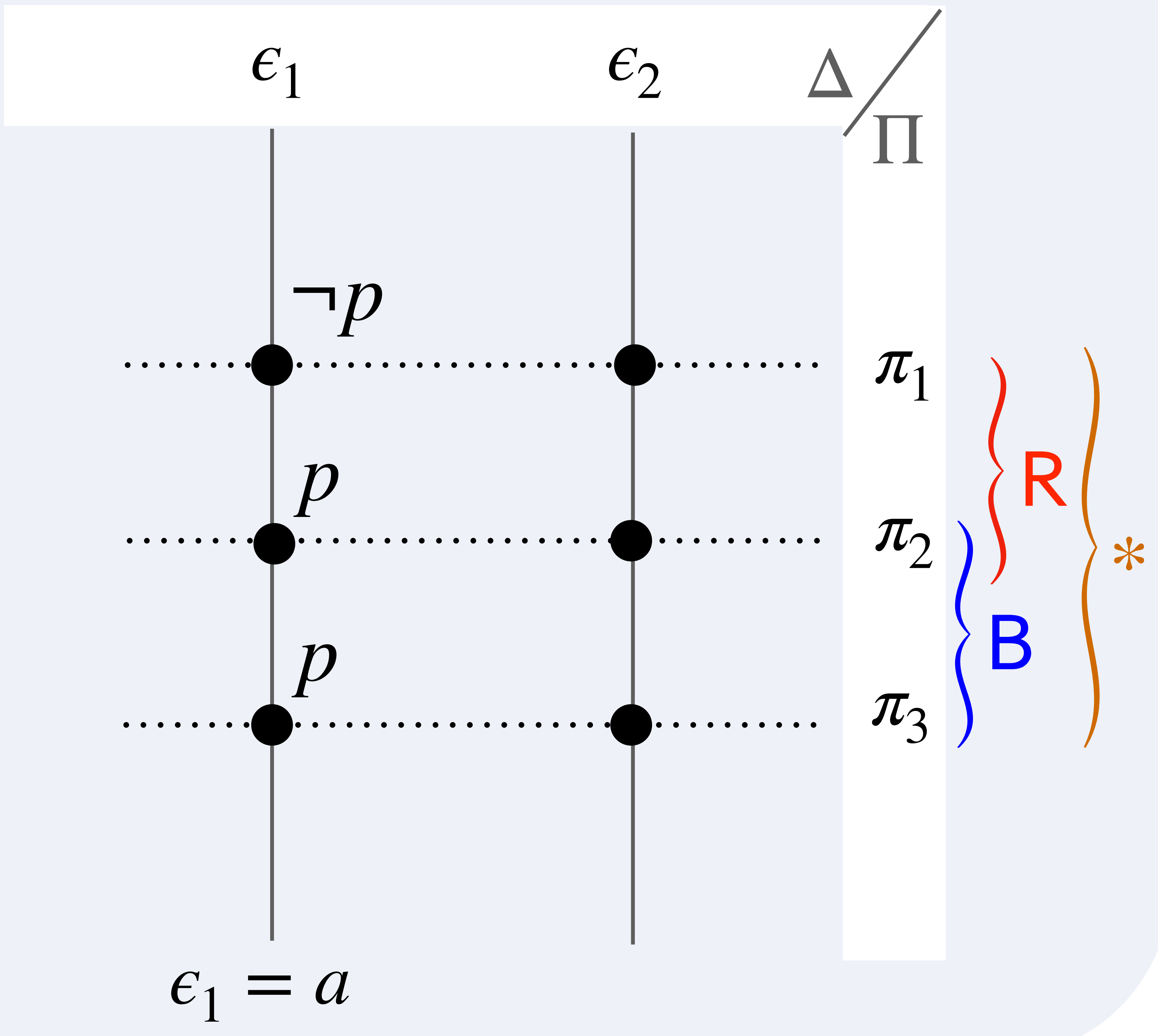
Relational semantics:

$$\mathscr{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathsf{B}) = \{\pi_2, \pi_3\}$
- $\gamma(\pi_1) = (p \mapsto \emptyset, \ldots)$

- $\mathscr{M} \models \Box_{\mathsf{B}}\, p(a)$

*Rigid domains and constants



First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

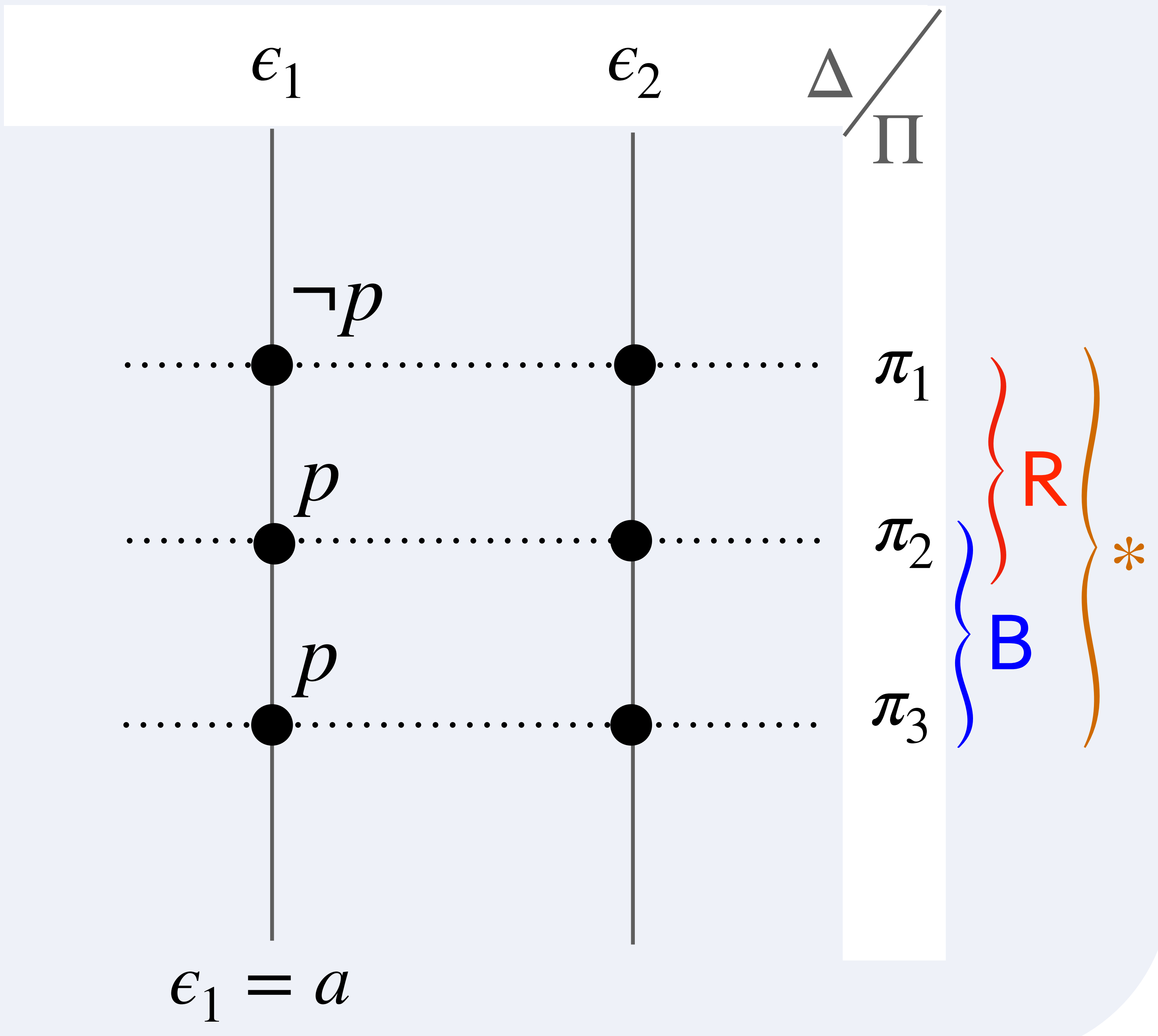
Relational semantics:

$$\mathcal{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathrm{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathrm{B}) = \{\pi_2, \pi_3\}$
- $\gamma(\pi_1) = (p \mapsto \emptyset, \ldots)$

- $\mathcal{M} \models \square_{\mathrm{B}}\, p(a)$
- $\mathcal{M} \models \lozenge_{\mathrm{R}} p(a) \wedge \lozenge_{\mathrm{R}} \neg p(a)$

*Rigid domains and constants



First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

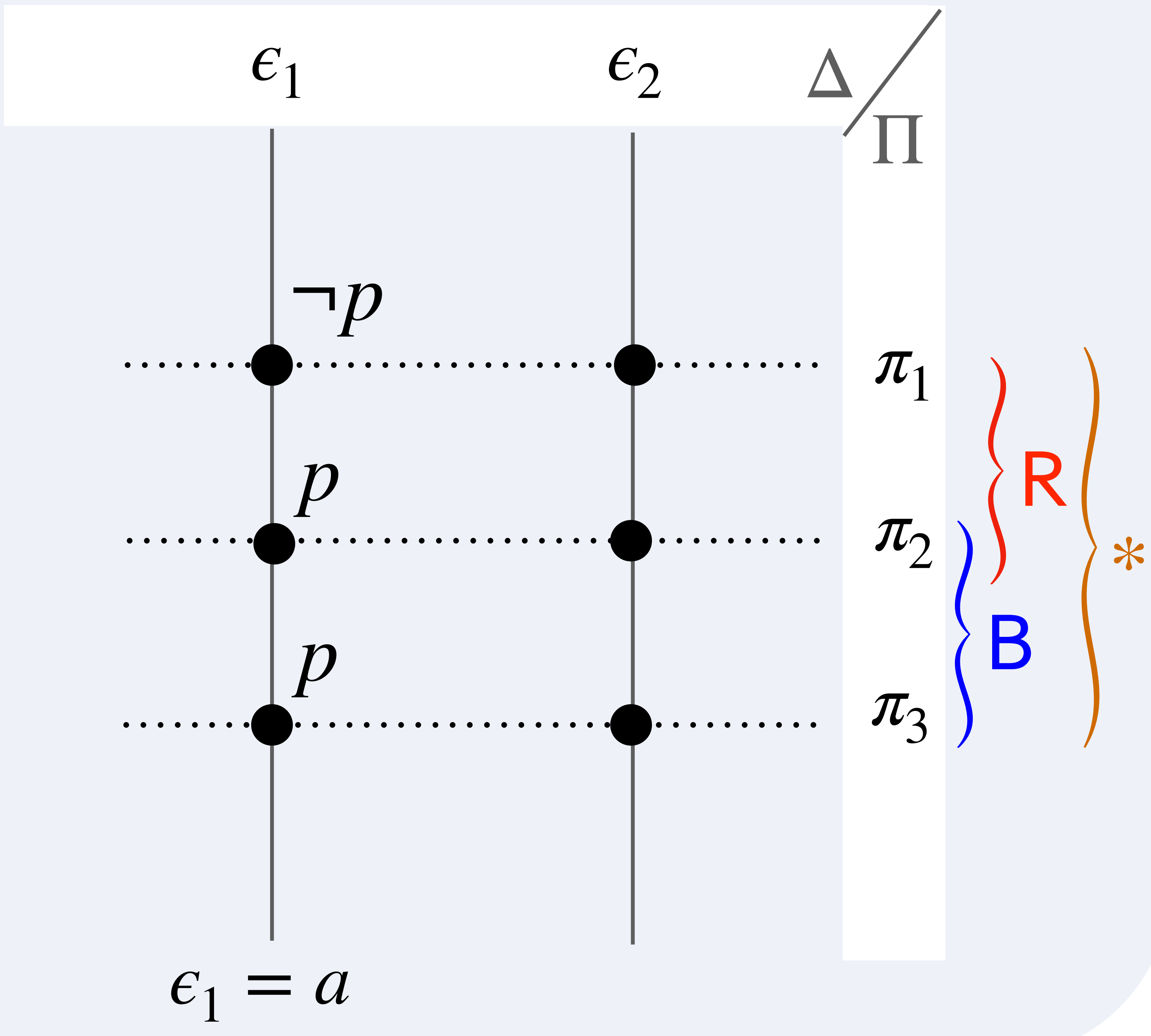
Relational semantics:

$$\mathcal{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathsf{B}) = \{\pi_2, \pi_3\}$
- $\gamma(\pi_1) = (p \mapsto \emptyset, \ldots)$

- $\mathcal{M} \models \square_{\mathsf{B}}\, p(a)$
- $\mathcal{M} \models \lozenge_{\mathsf{R}} p(a) \land \lozenge_{\mathsf{R}} \neg p(a)$
- $\mathcal{M} \models \square_{\mathsf{R}} \forall x\, p(x) \to (\exists y\; r(x, y))$

*Rigid domains and constants



First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

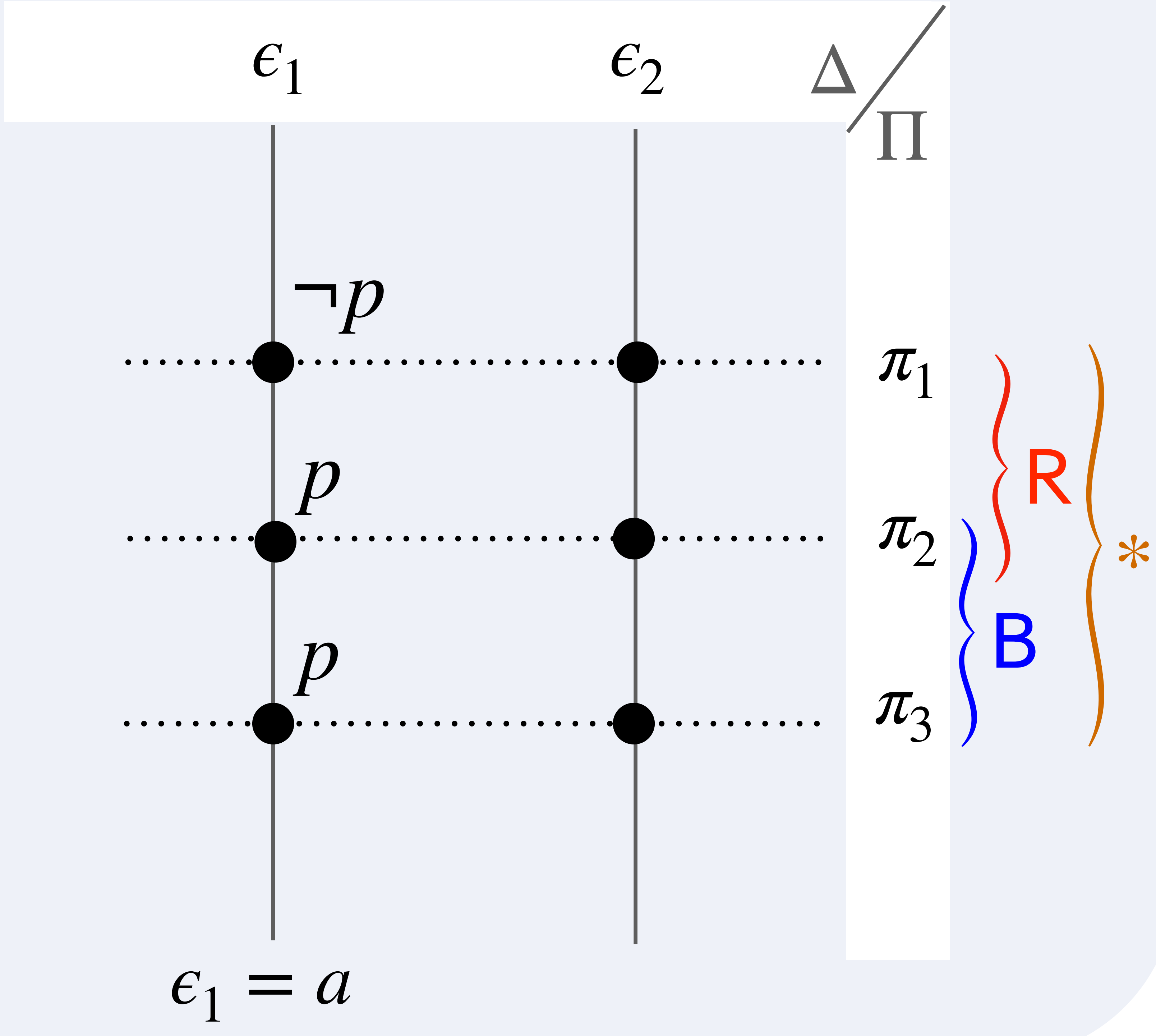
Relational semantics:

$$\mathcal{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathsf{B}) = \{\pi_2, \pi_3\}$
- $\gamma(\pi_1) = (p \mapsto \emptyset, \dots)$

- $\mathcal{M} \models \Box_{\mathsf{B}}\, p(a)$
- $\mathcal{M} \models \Diamond_{\mathsf{R}} p(a) \wedge \Diamond_{\mathsf{R}} \neg p(a)$
- $\mathcal{M} \models \Box_{\mathsf{R}} \forall x\, p(x) \rightarrow (\exists y\, r(x, y))$

*Rigid domains and constants



First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

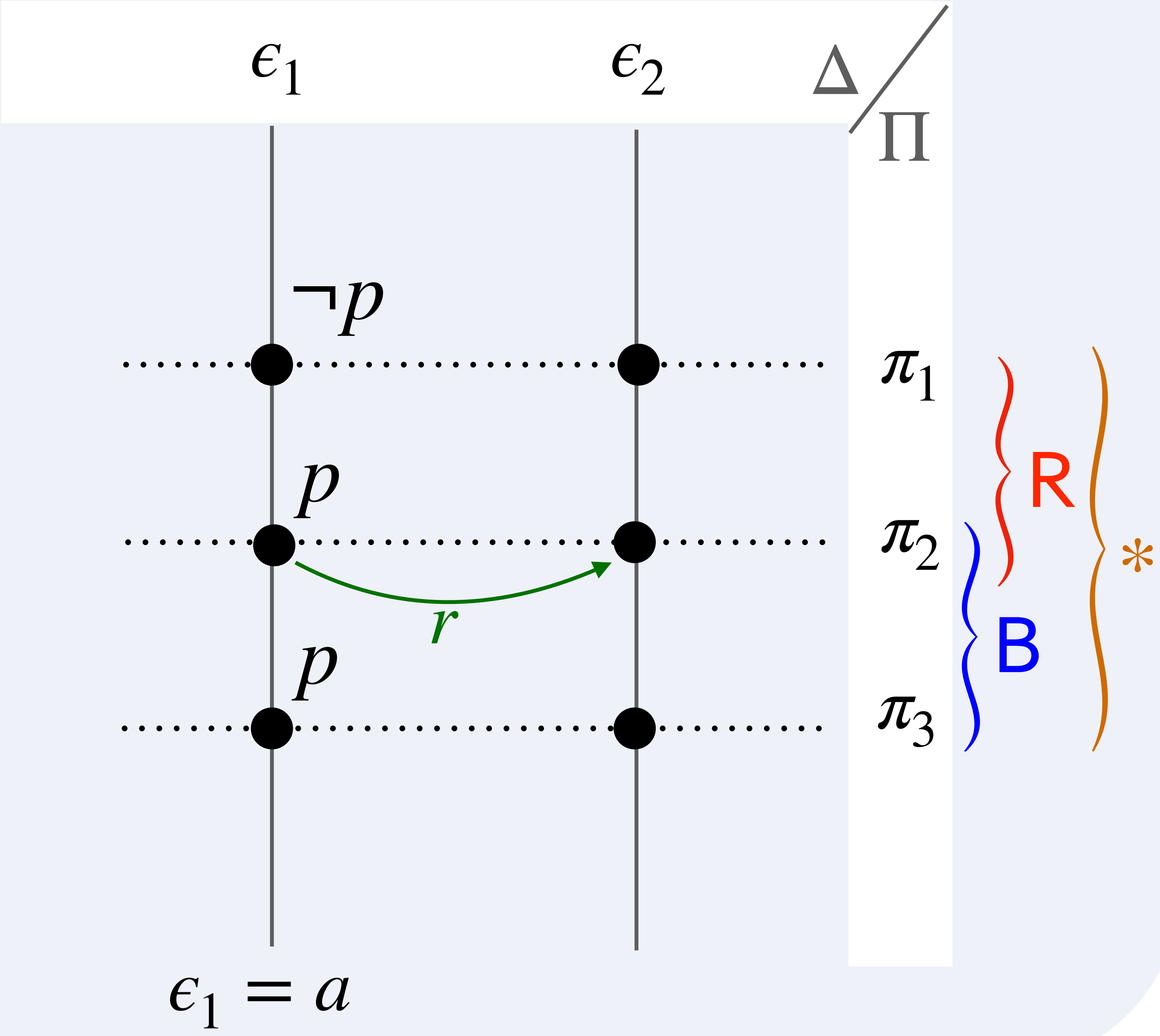
Relational semantics:

$$\mathcal{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathsf{B}) = \{\pi_2, \pi_3\}$
- $\gamma(\pi_1) = (p \mapsto \emptyset, \ldots)$

- $\mathcal{M} \models \Box_{\mathsf{B}}\, p(a)$
- $\mathcal{M} \models \Diamond_{\mathsf{R}} p(a) \wedge \Diamond_{\mathsf{R}} \neg p(a)$
- $\mathcal{M} \models \Box_{\mathsf{R}} \forall x\, p(x) \rightarrow (\exists y\; r(x, y))$

*Rigid domains and constants



First-Order Standpoint Logic: Semantics

Semantics of $\mathbb{S}$

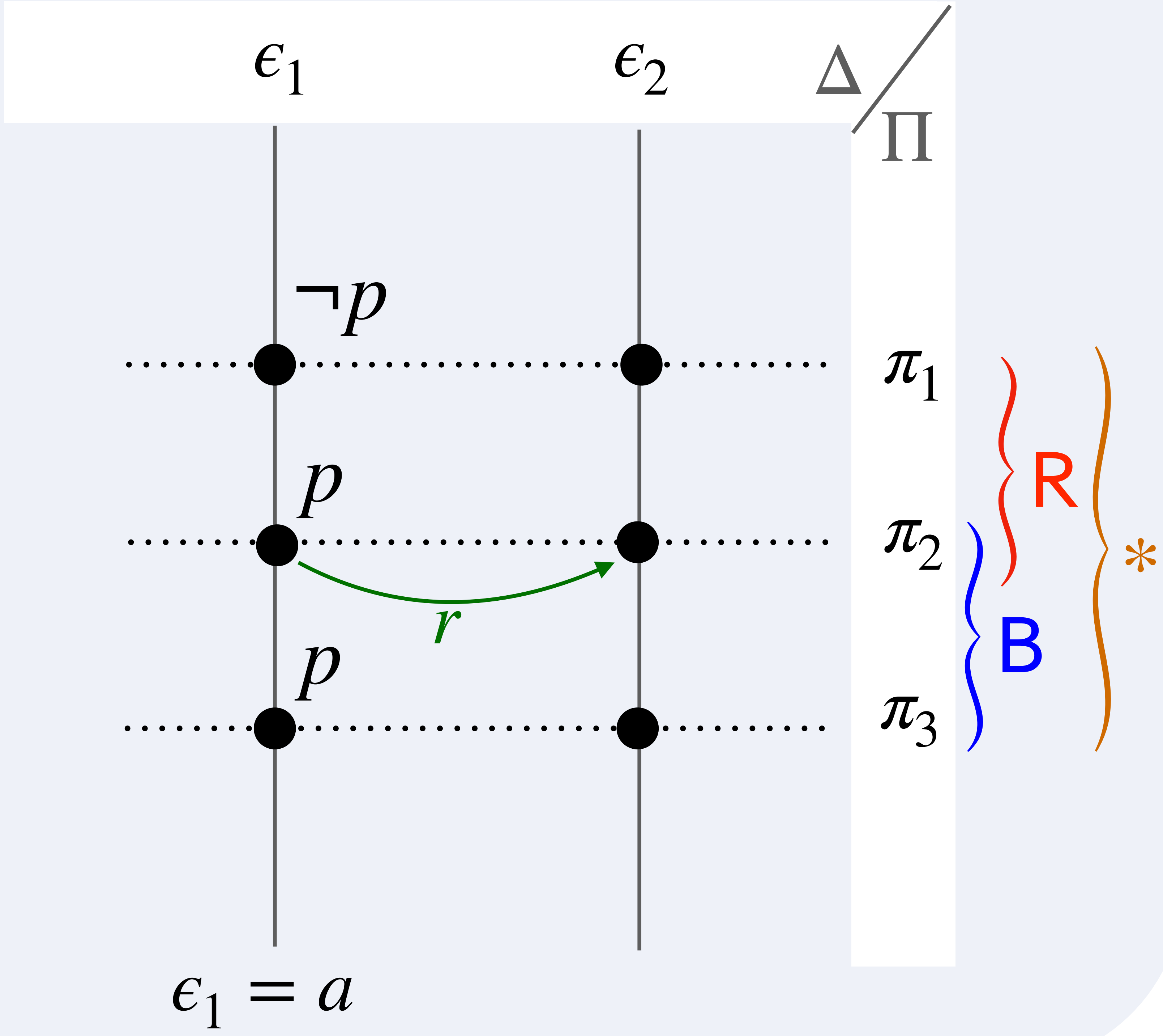
Relational semantics:

$$\mathcal{M} = \langle \Delta, \Pi, \sigma, \gamma \rangle$$

- $\Delta = \{\epsilon_1, \epsilon_2\}$
- $\Pi = \{\pi_1, \pi_2, \pi_3\}$
- $\sigma(\mathsf{R}) = \{\pi_1, \pi_2\}$
- $\sigma(\mathsf{B}) = \{\pi_2, \pi_3\}$
- $\gamma(\pi_1) = (p \mapsto \varnothing, \ldots)$

- $\mathcal{M} \models \Box_{\mathsf{B}}\, p(a)$
- $\mathcal{M} \models \Diamond_{\mathsf{R}} p(a) \wedge \Diamond_{\mathsf{R}} \neg p(a)$
- $\mathcal{M} \models \Box_{\mathsf{R}} \forall x\; p(x) \rightarrow (\exists y\; r(x, y))$

*Rigid domains and constants



Remark: The semantics of standpoint logic can also be expressed in standard Kripke (relational) semantics.

Extended Example

Knowledge Integration - Standpoint Logic

1. $\square_{\mathsf{S}}(\neg\exists x\ \mathsf{Process}(x) \land \mathsf{Tissue}(x))$
2. $\square_{\mathsf{H}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_{\mathsf{L}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \ \land\ \mathsf{H} \preceq \mathsf{S}$

Knowledge Integration - Standpoint Logic

1. $\square_{\mathsf{S}}(\neg \exists x\ \mathsf{Process}(x) \land \mathsf{Tissue}(x))$
2. $\square_{\mathsf{H}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_{\mathsf{L}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \ \land \ \mathsf{H} \preceq \mathsf{S}$

Knowledge Integration - Standpoint Logic

1. $\square_{\mathsf{S}}(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_{\mathsf{H}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_{\mathsf{L}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \Diamond_{\mathsf{S}}(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \square_{\mathsf{L}}\ \mathsf{Tumor}(x)$

Knowledge Integration - Standpoint Logic

1. $\square_{S}(\neg \exists x\ \mathrm{Process}(x) \wedge \mathrm{Tissue}(x))$
2. $\square_{H}(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Process}(x))$
3. $\square_{L}(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
4. $\mathrm{L} \preceq \mathrm{S} \ \wedge\ \mathrm{H} \preceq \mathrm{S}$
5. $\forall x\ \Diamond_{S}(\mathrm{Tumor}(x) \wedge \mathrm{Object}(x)) \rightarrow \square_{L}\ \mathrm{Tumor}(x)$
6. $\forall x\ \square_{H}(\exists y\ \mathrm{productOf}(x, y) \wedge \mathrm{Tumor}(y)) \leftrightarrow \Diamond_{L}\mathrm{Tumor}(x)$

Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg \exists x\ \mathrm{Process}(x) \land \mathrm{Tissue}(x))$
2. $\Box_H(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Process}(x))$
3. $\Box_L(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
4. $\mathrm{L} \preceq \mathrm{S}\ \land\ \mathrm{H} \preceq \mathrm{S}$
5. $\forall x\ \Diamond_S(\mathrm{Tumor}(x) \land \mathrm{Object}(x)) \rightarrow \Box_L\ \mathrm{Tumor}(x)$
6. $\forall x\ \Box_H\ (\exists y\ \mathrm{productOf}(x, y) \land \mathrm{Tumor}(y)) \leftrightarrow \Diamond_L \mathrm{Tumor}(x)$
7. $\forall x\ \Box_L\ \mathrm{Tissue}(x) \rightarrow\ \Box_H\ \mathrm{Tissue}(x)$

Knowledge Integration - Standpoint Logic

1. $\square_S(\neg \exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \ \wedge \ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \lozenge_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \square_L \mathsf{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \mathsf{productOf}(x, y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \lozenge_L \mathsf{Tumor}(x)$
7. $\forall x\ \square_L \mathsf{Tissue}(x) \rightarrow \square_H \mathsf{Tissue}(x)$

8. $\lozenge_S(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1, a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \text{Process}(x) \land \text{Tissue}(x))$
2. $\square_H(\forall x\ \text{Tumor}(x) \rightarrow \text{Process}(x))$
3. $\square_L(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $\text{L} \preceq \text{S} \ \land\ \text{H} \preceq \text{S}$
5. $\forall x\ \lozenge_S(\text{Tumor}(x) \land \text{Object}(x)) \rightarrow \square_L\ \text{Tumor}(x)$
6. $\forall x\ \square_H\ (\exists y\ \text{productOf}(x,y) \land \text{Tumor}(y)) \leftrightarrow \lozenge_L \text{Tumor}(x)$
7. $\forall x\ \square_L\ \text{Tissue}(x) \rightarrow \square_H\ \text{Tissue}(x)$

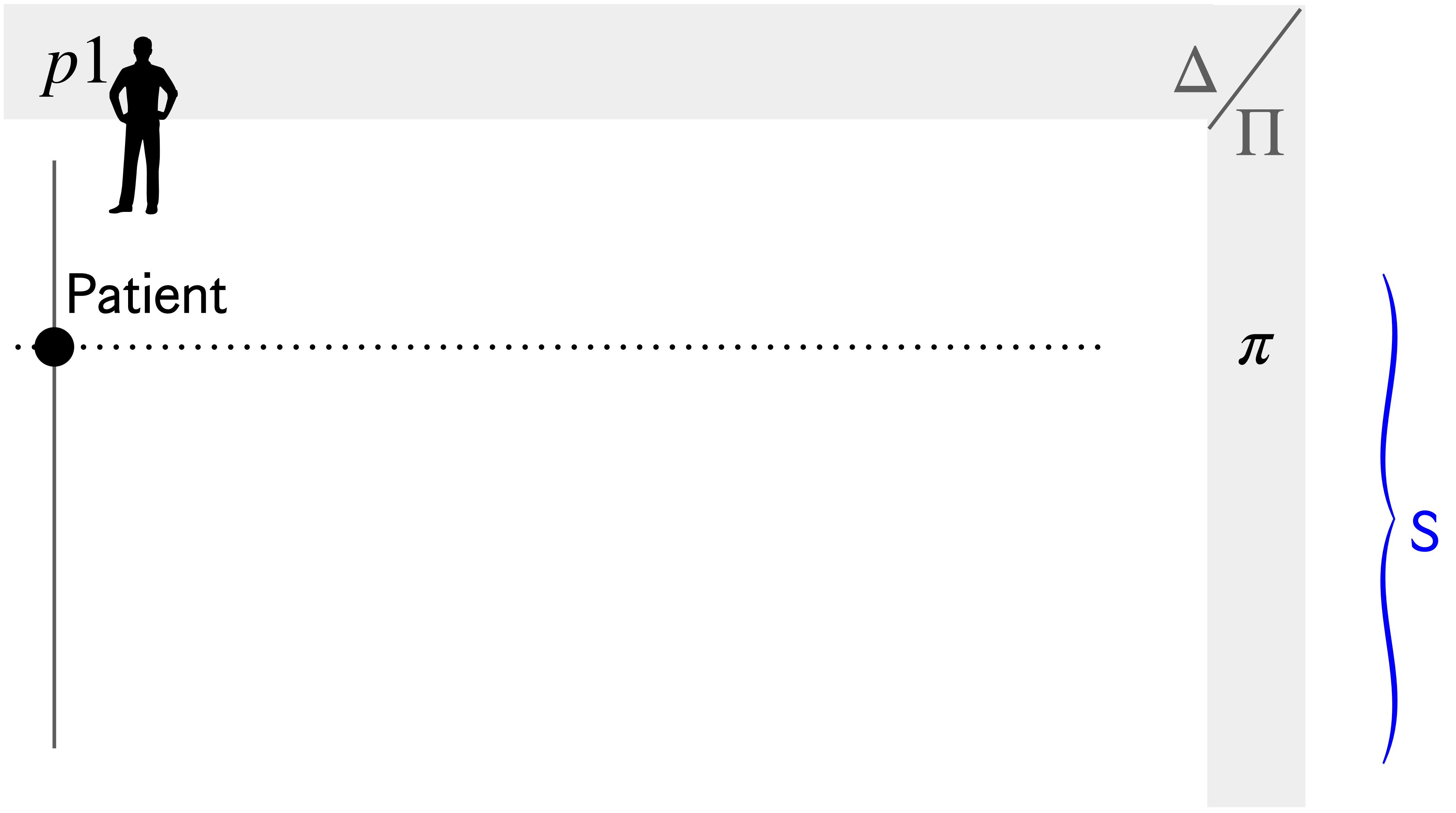
8. $\lozenge_S(\text{Patient}(p1) \land \text{HasPart}(p1,a) \land \text{Tumor}(a) \land \text{Object}(a))$

Δ / Π

Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \mathrm{Process}(x) \wedge \mathrm{Tissue}(x))$
2. $\square_H(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Process}(x))$
3. $\square_L(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
4. $L \preceq S \ \wedge\ H \preceq S$
5. $\forall x\ \lozenge_S(\mathrm{Tumor}(x) \wedge \mathrm{Object}(x)) \rightarrow \square_L\ \mathrm{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \mathrm{productOf}(x,y) \wedge \mathrm{Tumor}(y)) \leftrightarrow \lozenge_L \mathrm{Tumor}(x)$
7. $\forall x\ \square_L\ \mathrm{Tissue}(x) \rightarrow \square_H\ \mathrm{Tissue}(x)$

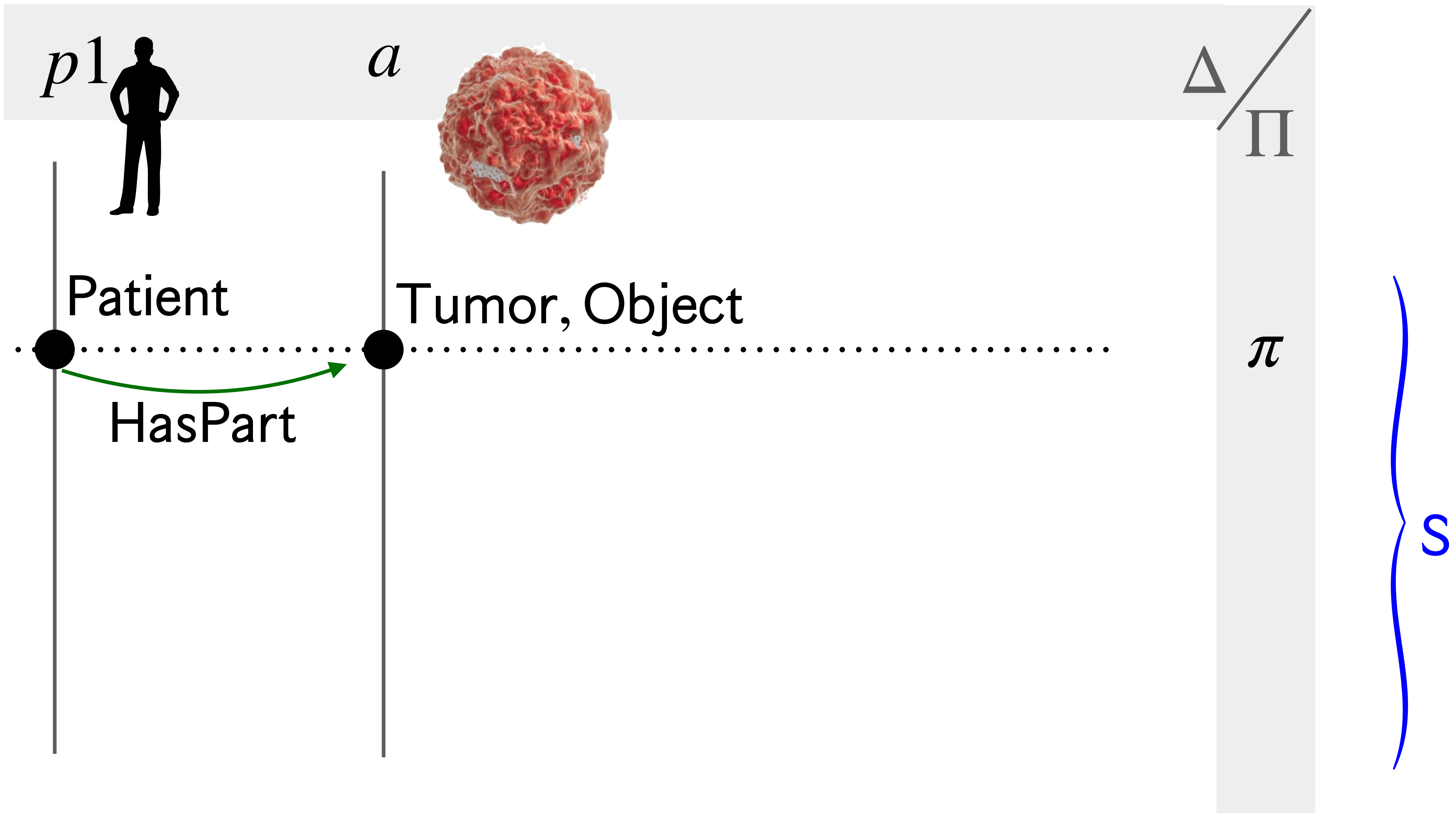
8. $\lozenge_S(\mathrm{Patient}(p1) \wedge \mathrm{HasPart}(p1,a) \wedge \mathrm{Tumor}(a) \wedge \mathrm{Object}(a))$



Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \text{Process}(x) \wedge \text{Tissue}(x))$
2. $\square_H(\forall x\ \text{Tumor}(x) \rightarrow \text{Process}(x))$
3. $\square_L(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $\text{L} \preceq \text{S} \ \wedge \ \text{H} \preceq \text{S}$
5. $\forall x\ \Diamond_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \square_L\ \text{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \text{productOf}(x,y) \wedge \text{Tumor}(y)) \leftrightarrow \Diamond_L\text{Tumor}(x)$
7. $\forall x\ \square_L\ \text{Tissue}(x) \rightarrow \square_H\ \text{Tissue}(x)$

8. $\Diamond_S(\text{Patient}(p1) \wedge \text{HasPart}(p1,a) \wedge \text{Tumor}(a) \wedge \text{Object}(a))$

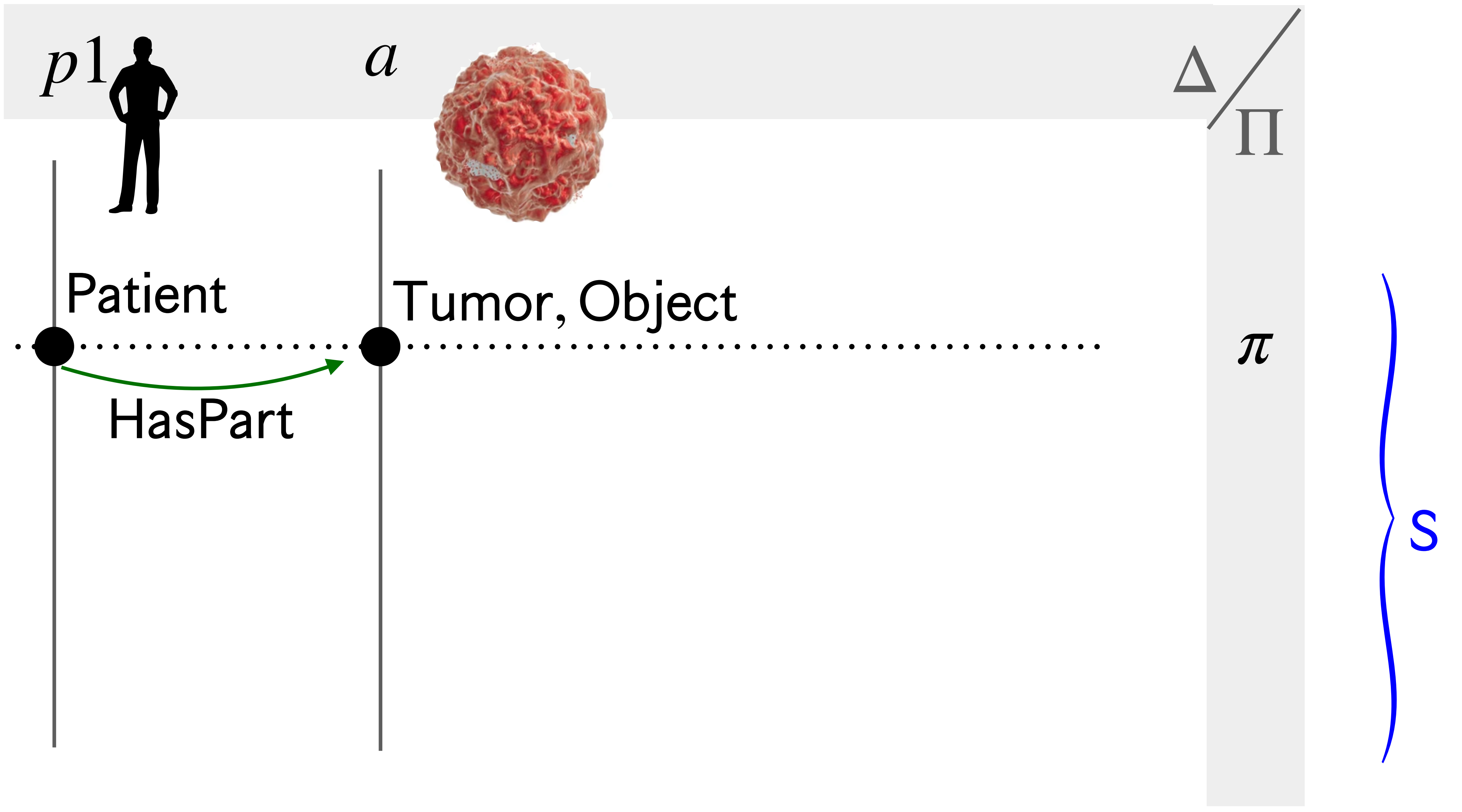


Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \text{Process}(x) \land \text{Tissue}(x))$
2. $\Box_H(\forall x\ \text{Tumor}(x) \rightarrow \text{Process}(x))$
3. $\Box_L(\forall x\ \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $L \preceq S \ \land\ H \preceq S$
5. $\forall x\ \Diamond_S(\text{Tumor}(x) \land \text{Object}(x)) \rightarrow \Box_L\ \text{Tumor}(x)$
6. $\forall x\ \Box_H(\exists y\ \text{productOf}(x,y) \land \text{Tumor}(y)) \leftrightarrow \Diamond_L \text{Tumor}(x)$
7. $\forall x\ \Box_L\ \text{Tissue}(x) \rightarrow \Box_H\ \text{Tissue}(x)$

8. $\Diamond_S(\text{Patient}(p1) \land \text{HasPart}(p1,a) \land \text{Tumor}(a) \land \text{Object}(a))$

Inferences:



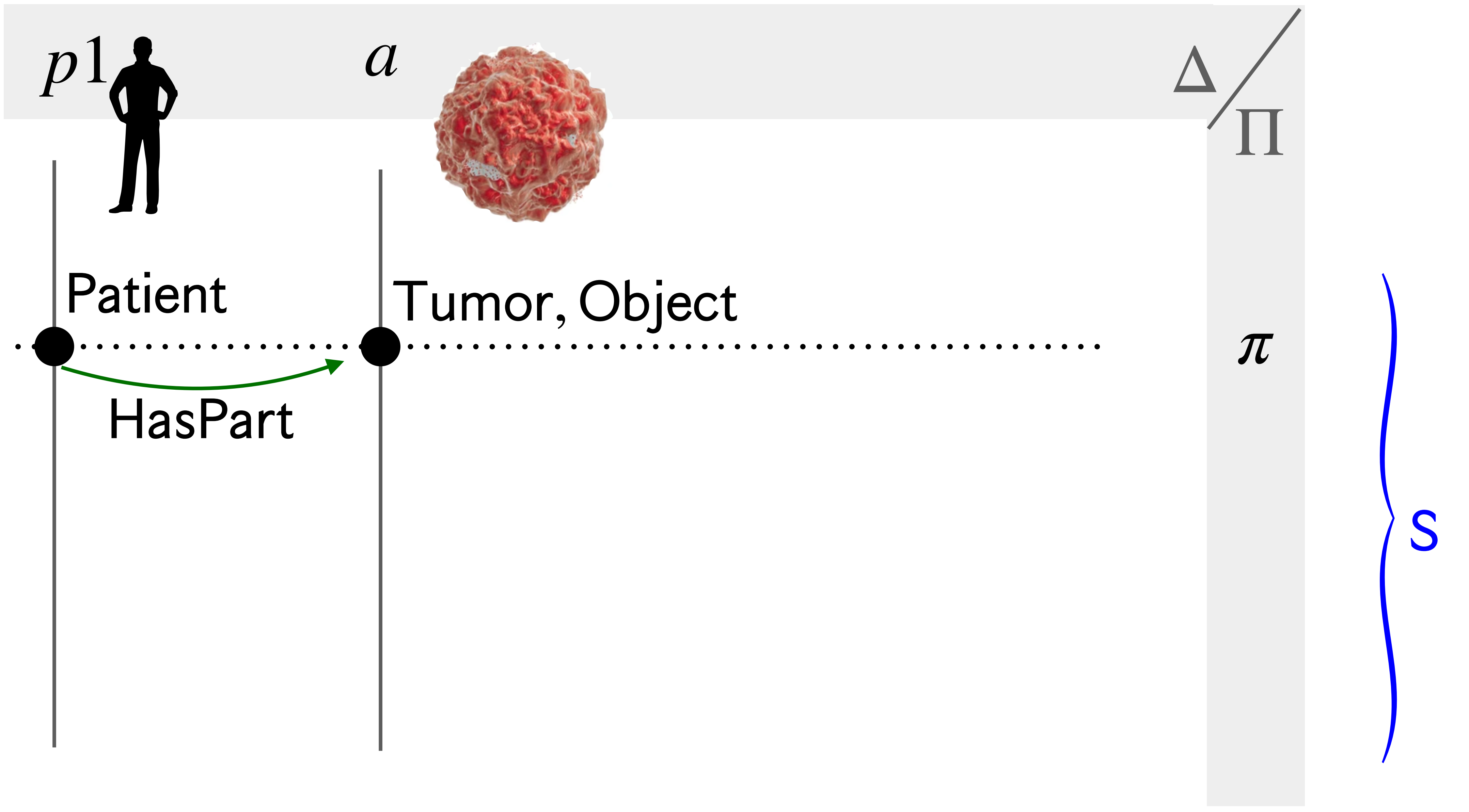
Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\square_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \lozenge_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \square_L\ \mathsf{Tumor}(x)$
6. $\forall x\ \square_H(\exists y\ \mathsf{productOf}(x,y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \lozenge_L \mathsf{Tumor}(x)$
7. $\forall x\ \square_L\ \mathsf{Tissue}(x) \rightarrow \square_H\ \mathsf{Tissue}(x)$

8. $\lozenge_S(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

9. $\square_L \mathsf{Tumor}(a)$ (5,8)



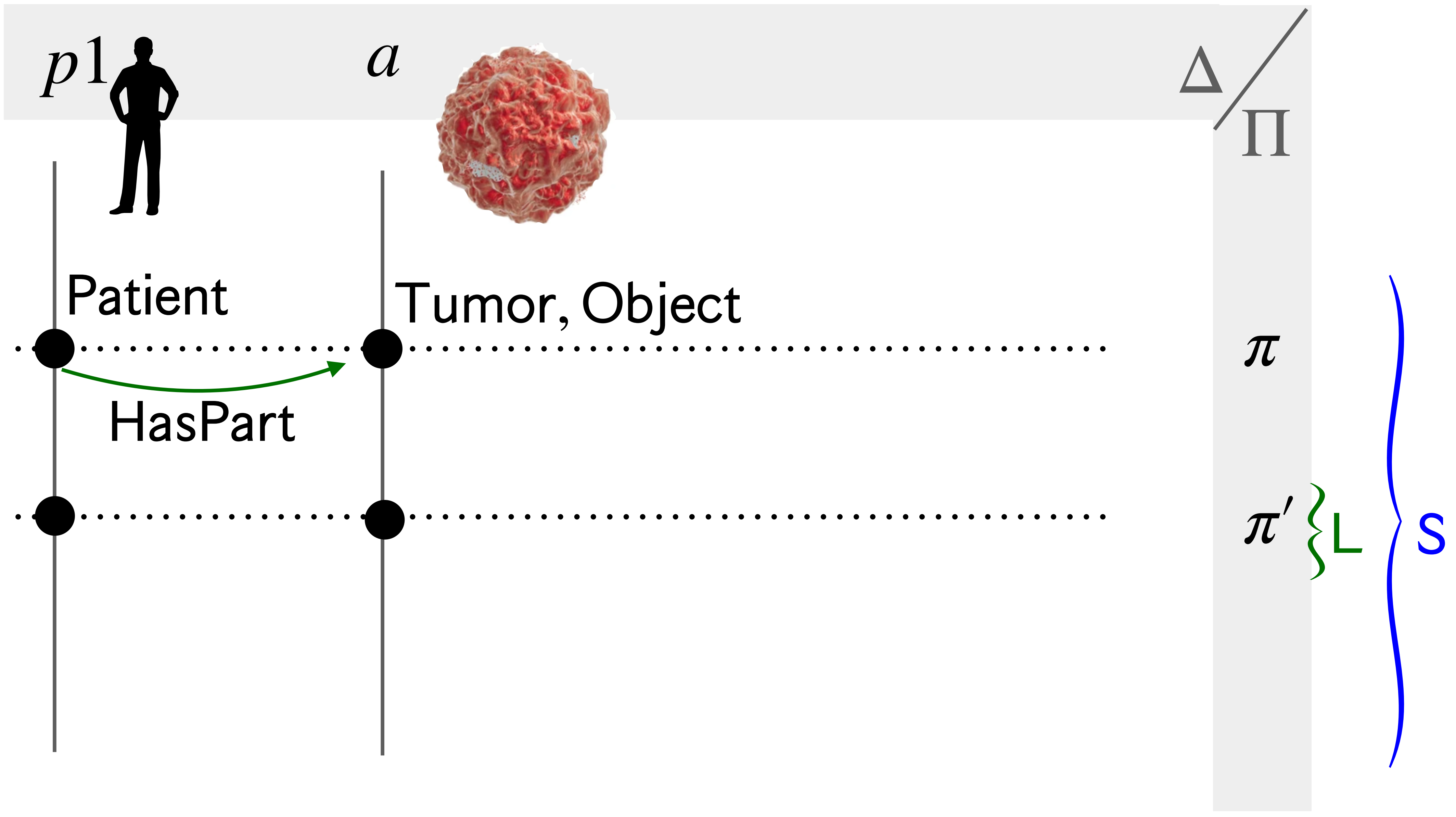
Knowledge Integration - Standpoint Logic

1. $\square_{S}(\neg \exists x\ \mathrm{Process}(x) \land \mathrm{Tissue}(x))$
2. $\square_{H}(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Process}(x))$
3. $\square_{L}(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
4. $L \preceq S \ \land \ H \preceq S$
5. $\forall x\ \lozenge_{S}(\mathrm{Tumor}(x) \land \mathrm{Object}(x)) \rightarrow \square_{L}\ \mathrm{Tumor}(x)$
6. $\forall x\ \square_{H}(\exists y\ \mathrm{productOf}(x, y) \land \mathrm{Tumor}(y)) \leftrightarrow \lozenge_{L}\mathrm{Tumor}(x)$
7. $\forall x\ \square_{L}\ \mathrm{Tissue}(x) \rightarrow \square_{H}\ \mathrm{Tissue}(x)$

8. $\lozenge_{S}(\mathrm{Patient}(p1) \land \mathrm{HasPart}(p1,a) \land \mathrm{Tumor}(a) \land \mathrm{Object}(a))$

Inferences:

9. $\square_{L}\mathrm{Tumor}(a)$ (5,8)



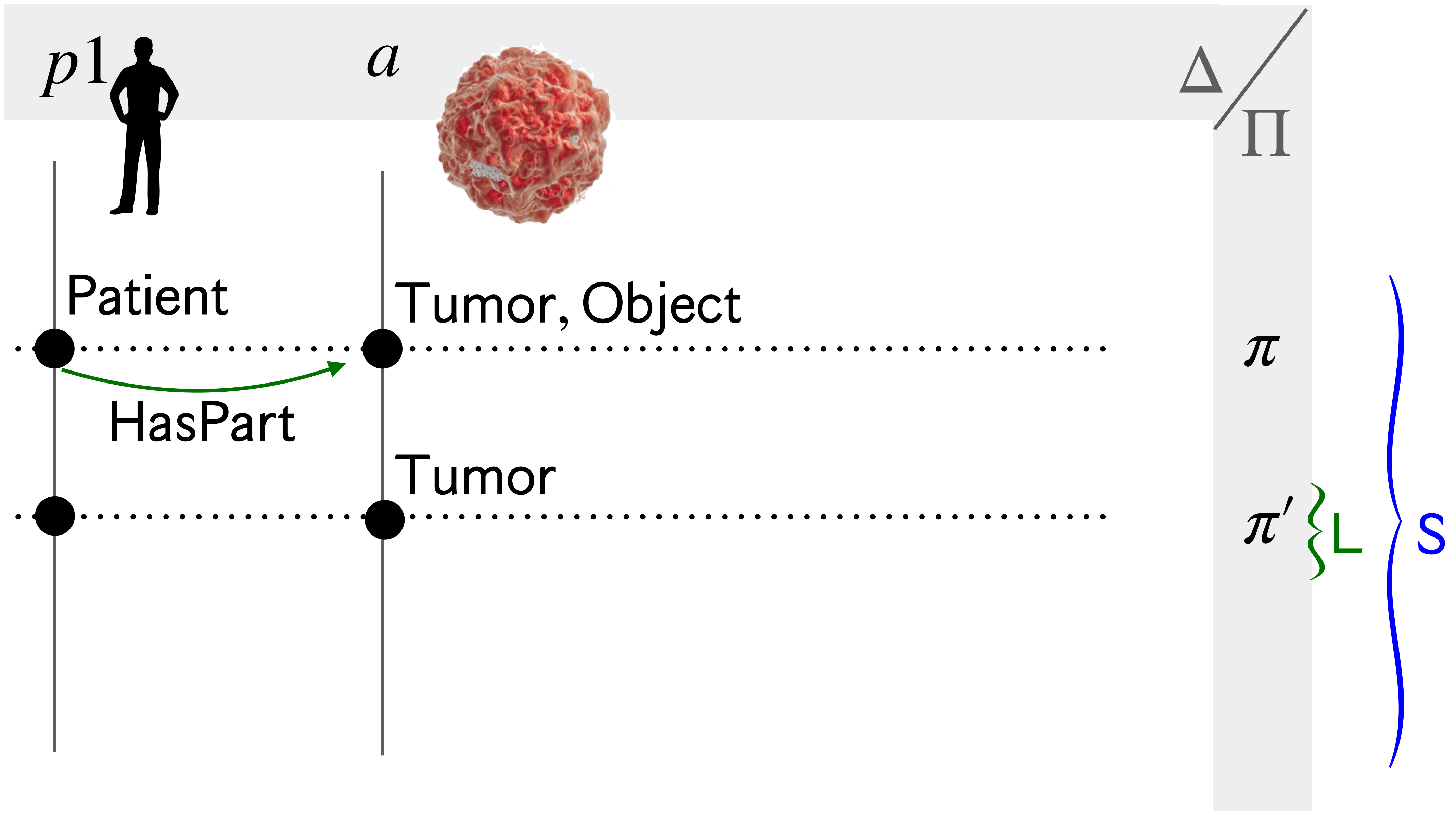
Knowledge Integration - Standpoint Logic

1. $\square_S(\neg\exists x\ \mathsf{Process}(x) \land \mathsf{Tissue}(x))$
2. $\square_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \land\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \Diamond_S(\mathsf{Tumor}(x) \land \mathsf{Object}(x)) \rightarrow \square_L\, \mathsf{Tumor}(x)$
6. $\forall x\ \square_H\,(\exists y\ \mathsf{productOf}(x,y) \land \mathsf{Tumor}(y)) \leftrightarrow \Diamond_L \mathsf{Tumor}(x)$
7. $\forall x\ \square_L\, \mathsf{Tissue}(x) \rightarrow \square_H\, \mathsf{Tissue}(x)$

8. $\Diamond_S(\mathsf{Patient}(p1) \land \mathsf{HasPart}(p1,a) \land \mathsf{Tumor}(a) \land \mathsf{Object}(a))$

Inferences:

9. $\square_L \mathsf{Tumor}(a)$ (5,8)



Knowledge Integration - Standpoint Logic

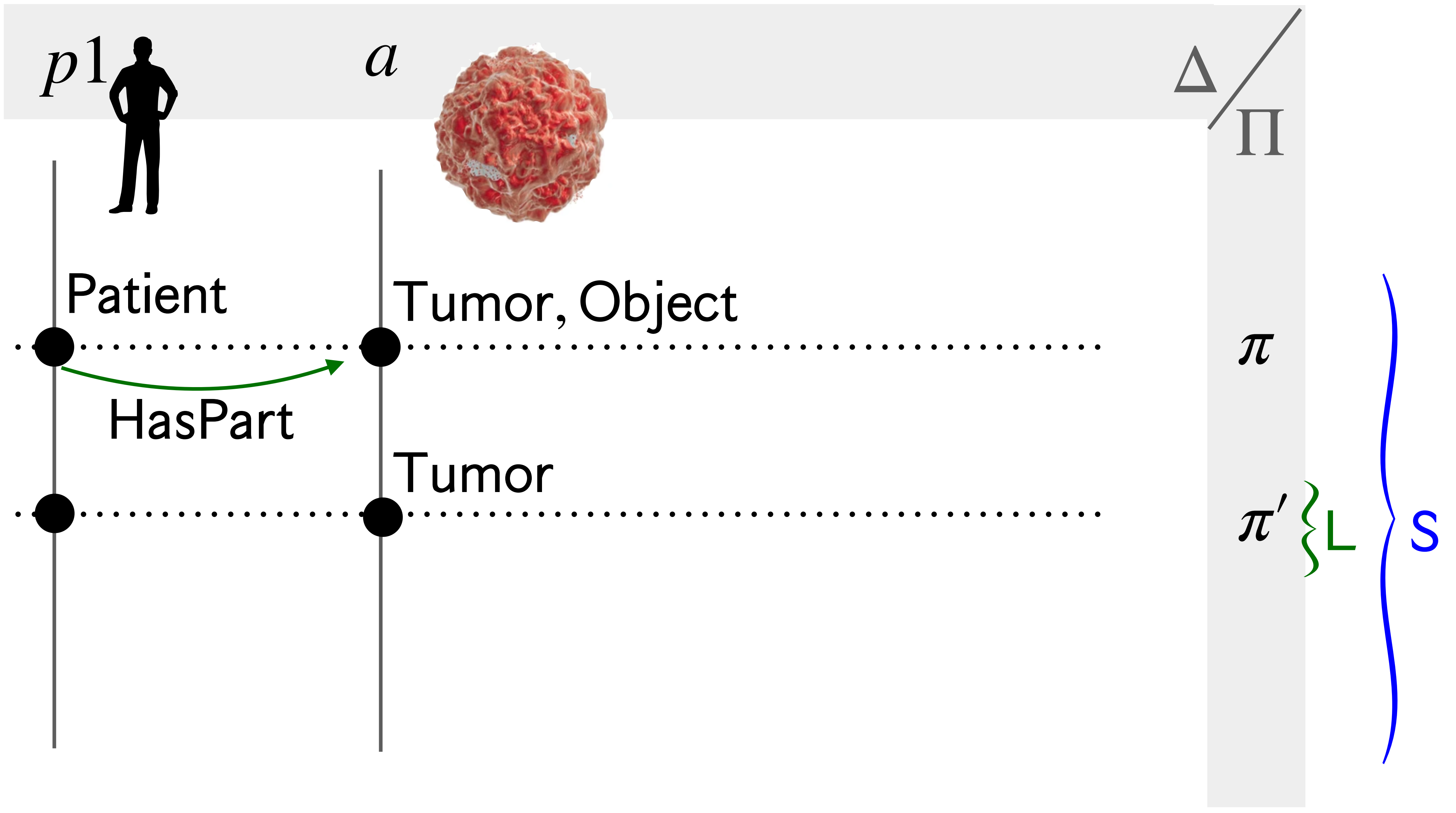
1. $\Box_S(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\Box_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\Box_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \Diamond_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \Box_L\, \mathsf{Tumor}(x)$
6. $\forall x\ \Box_H\,(\exists y\ \mathsf{productOf}(x,y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \Diamond_L \mathsf{Tumor}(x)$
7. $\forall x\ \Box_L\, \mathsf{Tissue}(x) \rightarrow \Box_H\, \mathsf{Tissue}(x)$

8. $\Diamond_S(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

9. $\Box_L \mathsf{Tumor}(a)$ (5,8)

10. $\Box_L \mathsf{Tissue}(a)$ (9,3)



Knowledge Integration - Standpoint Logic

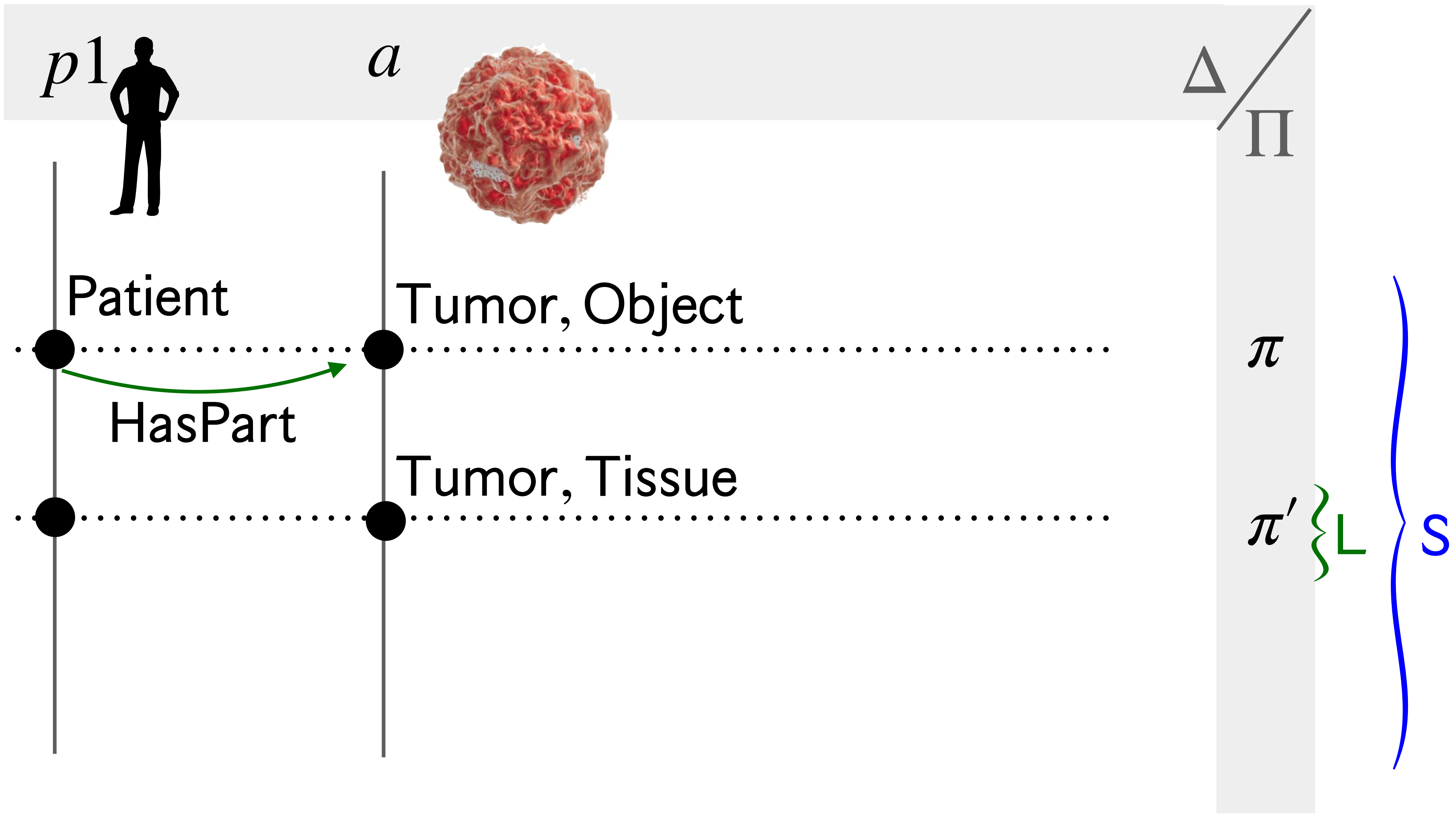
1. $\square_S(\neg \exists x \; \text{Process}(x) \wedge \text{Tissue}(x))$
2. $\square_H(\forall x \; \text{Tumor}(x) \rightarrow \text{Process}(x))$
3. $\square_L(\forall x \; \text{Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $L \preceq S \;\wedge\; H \preceq S$
5. $\forall x \; \lozenge_S(\text{Tumor}(x) \wedge \text{Object}(x)) \rightarrow \square_L \text{Tumor}(x)$
6. $\forall x \; \square_H (\exists y \; \text{productOf}(x, y) \wedge \text{Tumor}(y)) \leftrightarrow \lozenge_L \text{Tumor}(x)$
7. $\forall x \; \square_L \text{Tissue}(x) \rightarrow \square_H \text{Tissue}(x)$

8. $\lozenge_S(\text{Patient}(p1) \wedge \text{HasPart}(p1,a) \wedge \text{Tumor}(a) \wedge \text{Object}(a))$

Inferences:

9. $\square_L \text{Tumor}(a)$ (5,8)

10. $\square_L \text{Tissue}(a)$ (9,3)



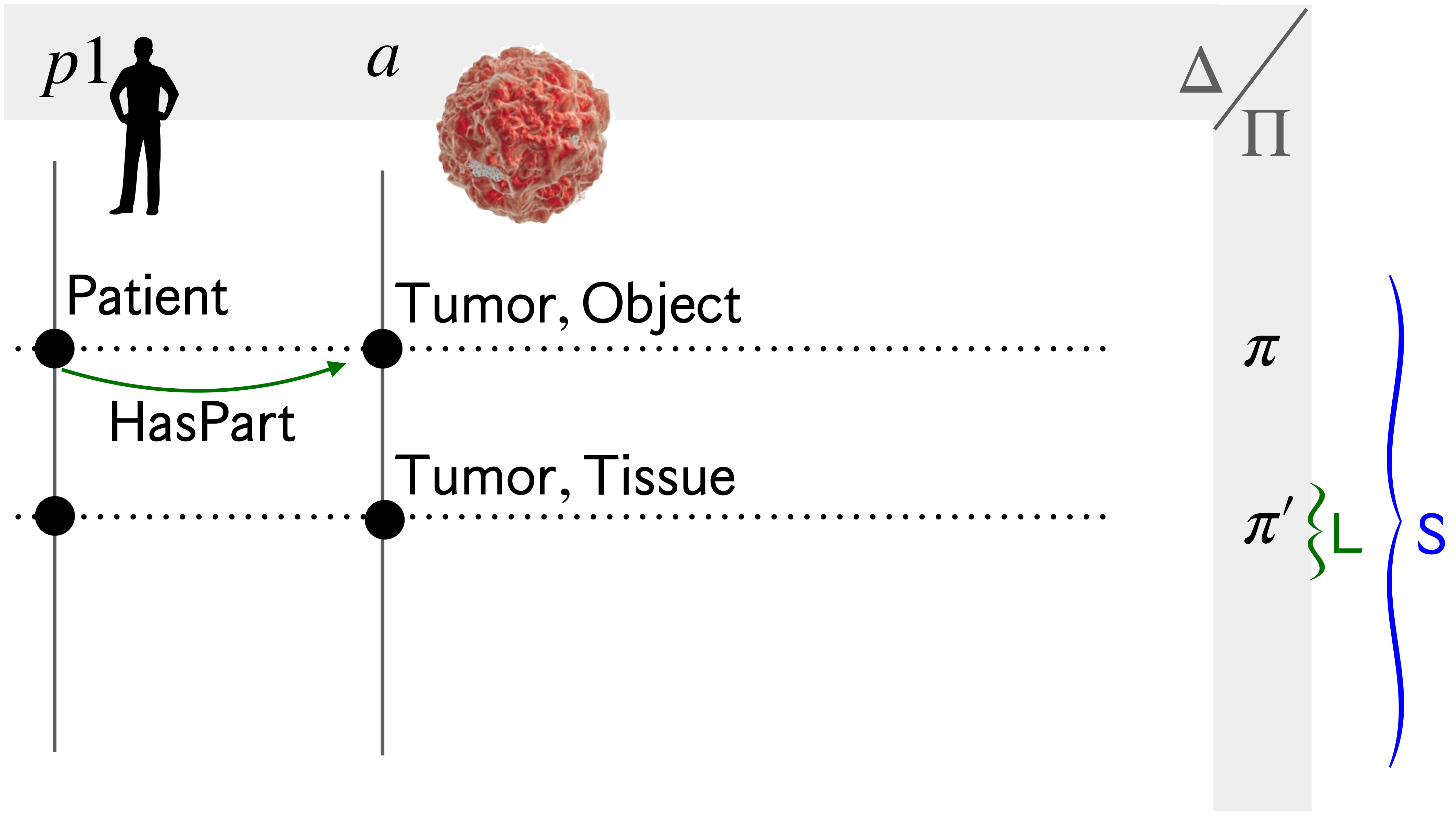
Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \mathrm{Process}(x) \land \mathrm{Tissue}(x))$
2. $\Box_H(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Process}(x))$
3. $\Box_L(\forall x\ \mathrm{Tumor}(x) \rightarrow \mathrm{Tissue}(x))$
4. $L \preceq S \ \land \ H \preceq S$
5. $\forall x\ \Diamond_S(\mathrm{Tumor}(x) \land \mathrm{Object}(x)) \rightarrow \Box_L\ \mathrm{Tumor}(x)$
6. $\forall x\ \Box_H\ (\exists y\ \mathrm{productOf}(x, y) \land \mathrm{Tumor}(y)) \leftrightarrow \Diamond_L \mathrm{Tumor}(x)$
7. $\forall x\ \Box_L\ \mathrm{Tissue}(x) \rightarrow \Box_H\ \mathrm{Tissue}(x)$

8. $\Diamond_S(\mathrm{Patient}(p1) \land \mathrm{HasPart}(p1,a) \land \mathrm{Tumor}(a) \land \mathrm{Object}(a))$

Inferences:

9. $\Box_L \mathrm{Tumor}(a)$ (5,8)
10. $\Box_L \mathrm{Tissue}(a)$ (9,3)
11. $\Box_H \mathrm{Tissue}(a)$ (10,7)



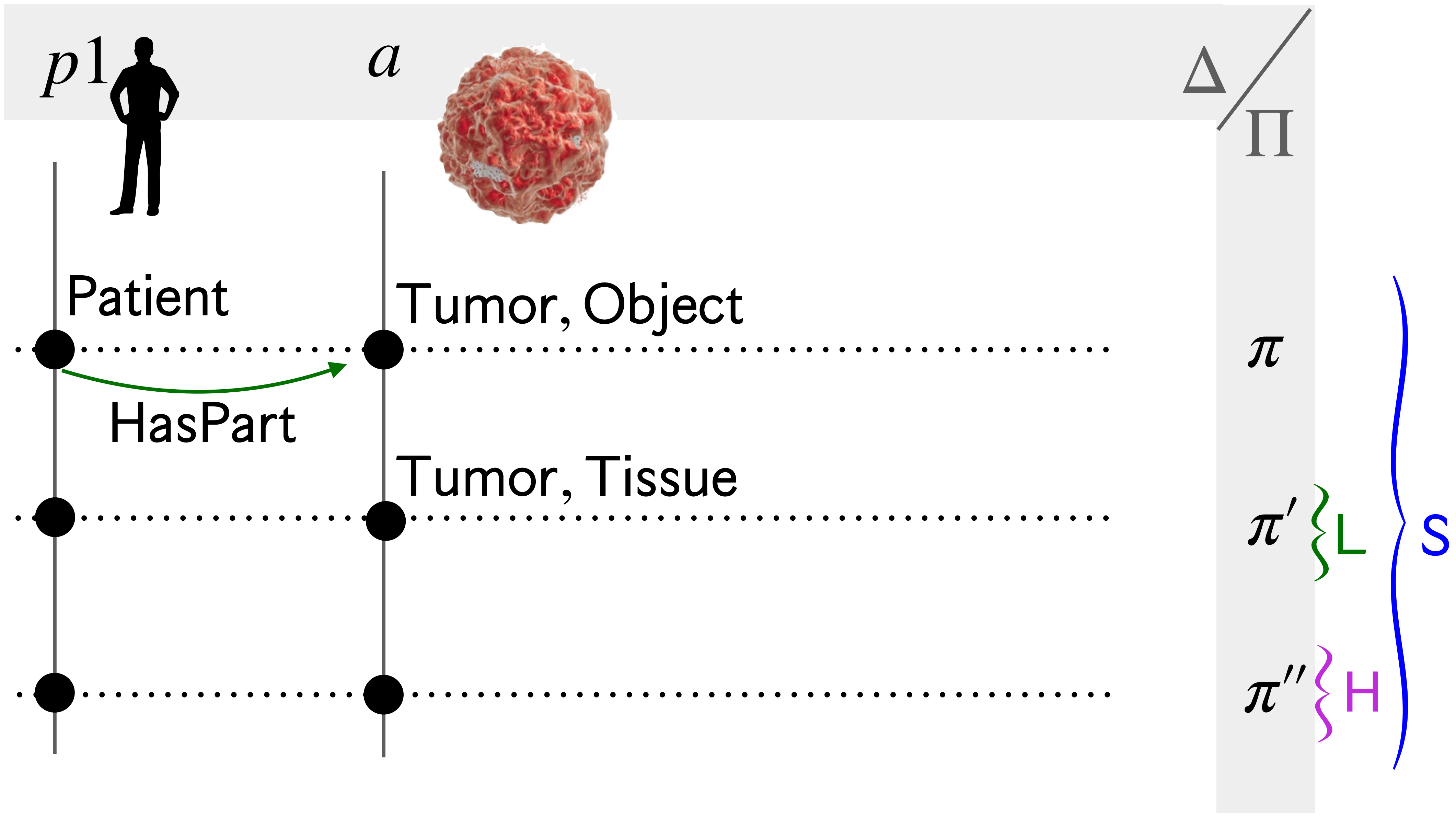
Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \mathsf{Process}(x) \land \mathsf{Tissue}(x))$
2. $\Box_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\Box_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \land\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \Diamond_S(\mathsf{Tumor}(x) \land \mathsf{Object}(x)) \rightarrow \Box_L\ \mathsf{Tumor}(x)$
6. $\forall x\ \Box_H\,(\exists y\ \mathsf{productOf}(x,y) \land \mathsf{Tumor}(y)) \leftrightarrow \Diamond_L \mathsf{Tumor}(x)$
7. $\forall x\ \Box_L\ \mathsf{Tissue}(x) \rightarrow \Box_H\ \mathsf{Tissue}(x)$

8. $\Diamond_S(\mathsf{Patient}(p1) \land \mathsf{HasPart}(p1,a) \land \mathsf{Tumor}(a) \land \mathsf{Object}(a))$

Inferences:

9. $\Box_L \mathsf{Tumor}(a)$ (5,8)
10. $\Box_L \mathsf{Tissue}(a)$ (9,3)
11. $\Box_H \mathsf{Tissue}(a)$ (10,7)



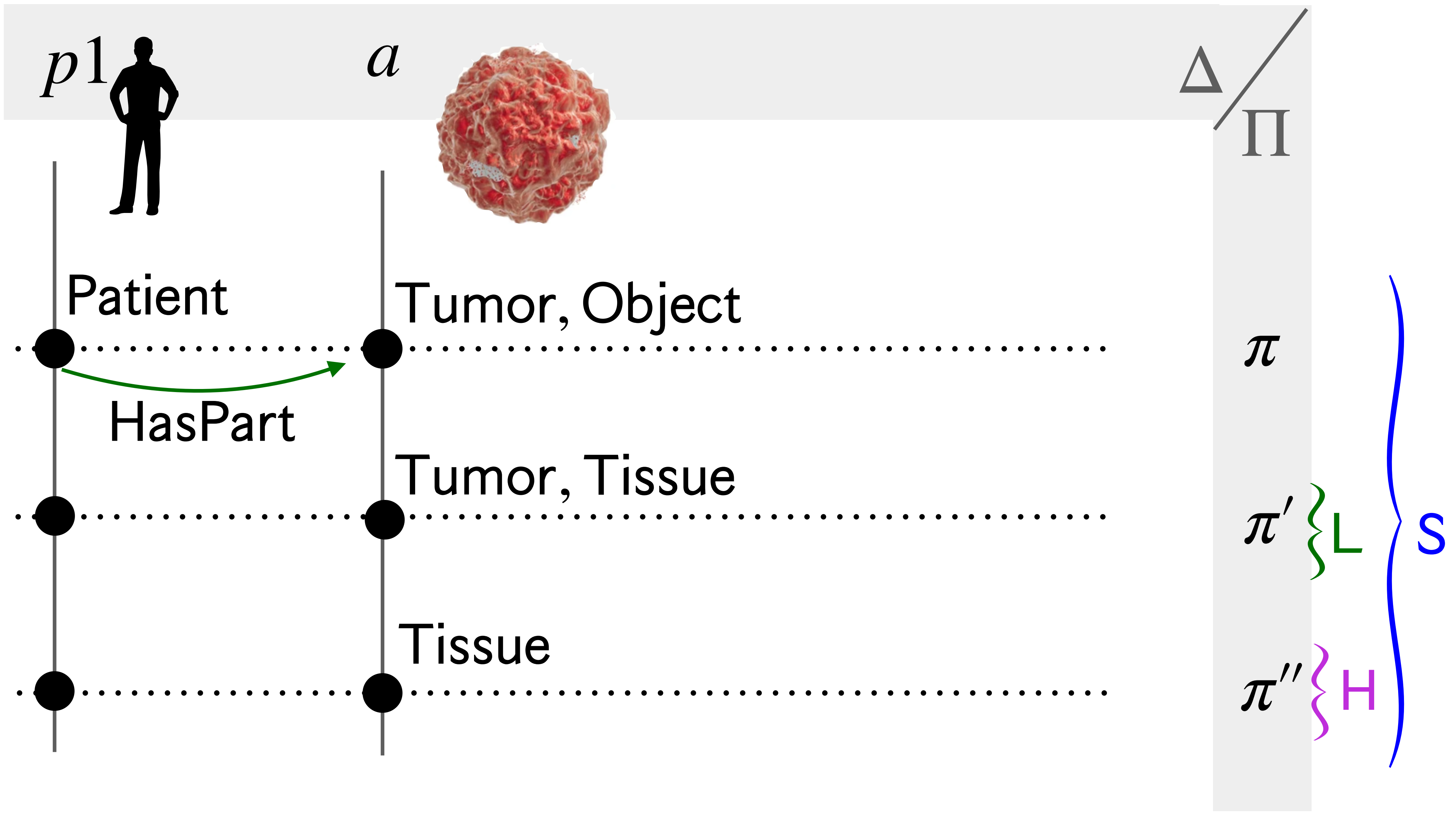
Knowledge Integration - Standpoint Logic

1. $\square_{\mathsf{S}}(\neg \exists x\ \mathsf{Process}(x) \land \mathsf{Tissue}(x))$
2. $\square_{\mathsf{H}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\square_{\mathsf{L}}(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \ \land\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \lozenge_{\mathsf{S}}(\mathsf{Tumor}(x) \land \mathsf{Object}(x)) \rightarrow \square_{\mathsf{L}}\, \mathsf{Tumor}(x)$
6. $\forall x\ \square_{\mathsf{H}}\,(\exists y\ \mathsf{productOf}(x,y) \land \mathsf{Tumor}(y)) \leftrightarrow \lozenge_{\mathsf{L}}\mathsf{Tumor}(x)$
7. $\forall x\ \square_{\mathsf{L}}\, \mathsf{Tissue}(x) \rightarrow \square_{\mathsf{H}}\, \mathsf{Tissue}(x)$

8. $\lozenge_{\mathsf{S}}(\mathsf{Patient}(p1) \land \mathsf{HasPart}(p1,a) \land \mathsf{Tumor}(a) \land \mathsf{Object}(a))$

Inferences:

9. $\square_{\mathsf{L}}\mathsf{Tumor}(a)$ (5,8)
10. $\square_{\mathsf{L}}\mathsf{Tissue}(a)$ (9,3)
11. $\square_{\mathsf{H}}\mathsf{Tissue}(a)$ (10,7)



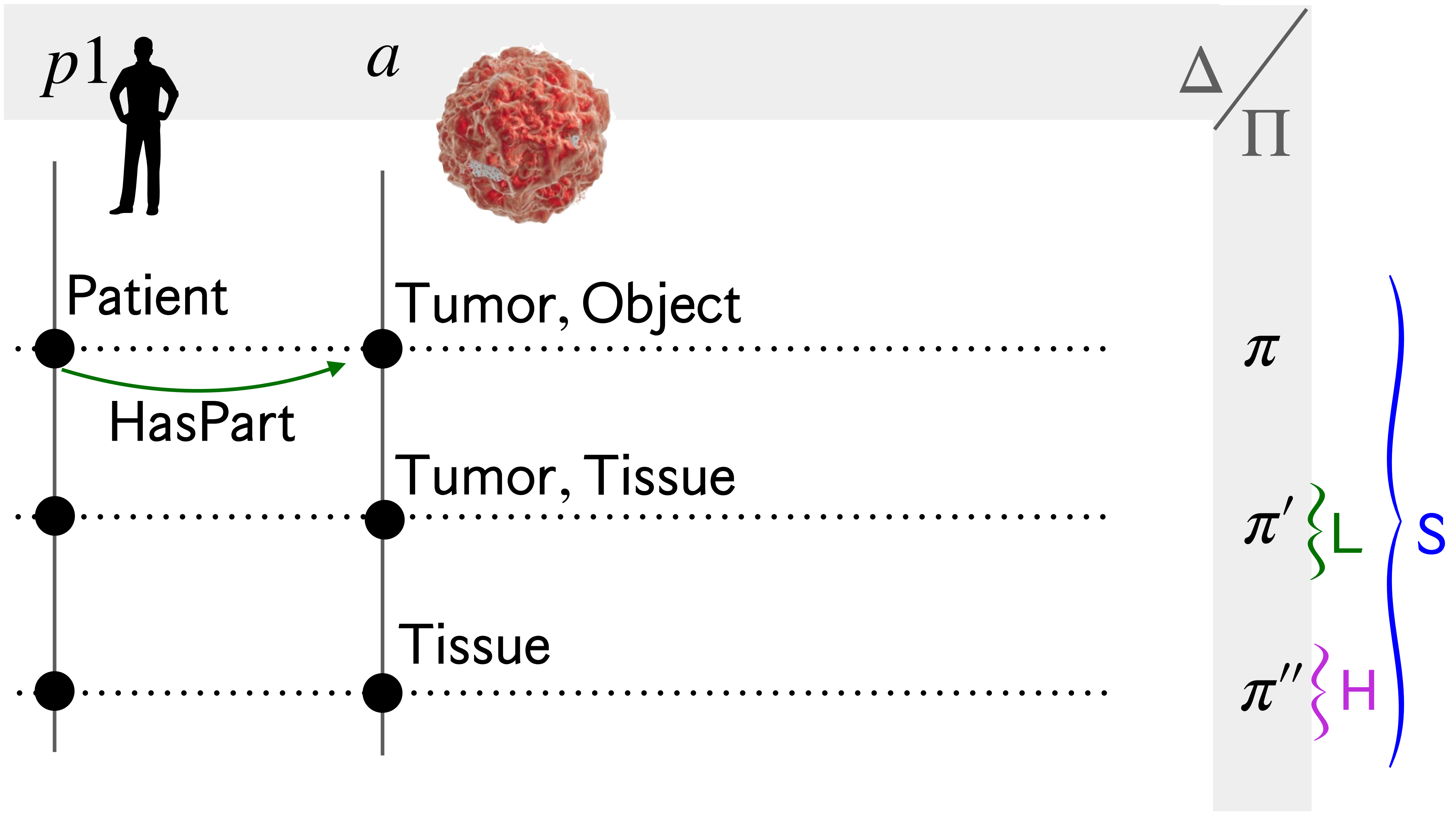
Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\Box_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\Box_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \ \wedge\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \Diamond_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \Box_L\,\mathsf{Tumor}(x)$
6. $\forall x\ \Box_H\,(\exists y\ \mathsf{productOf}(x,y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \Diamond_L\mathsf{Tumor}(x)$
7. $\forall x\ \Box_L\,\mathsf{Tissue}(x) \rightarrow \Box_H\,\mathsf{Tissue}(x)$

8. $\Diamond_S(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

9. $\Box_L\mathsf{Tumor}(a)$ (5,8)
10. $\Box_L\mathsf{Tissue}(a)$ (9,3)
11. $\Box_H\mathsf{Tissue}(a)$ (10,7)
12. $\Box_H\,\exists y\ \mathsf{productOf}(a,y) \wedge \mathsf{Tumor}(y)$ (11,6)



Knowledge Integration - Standpoint Logic

1. $\square_{S}(\neg \exists x \text{ Process}(x) \land \text{Tissue}(x))$
2. $\square_{H}(\forall x \text{ Tumor}(x) \rightarrow \text{Process}(x))$
3. $\square_{L}(\forall x \text{ Tumor}(x) \rightarrow \text{Tissue}(x))$
4. $L \preceq S \ \land \ H \preceq S$
5. $\forall x \ \lozenge_{S}(\text{Tumor}(x) \land \text{Object}(x)) \rightarrow \square_{L} \text{Tumor}(x)$
6. $\forall x \ \square_{H}(\exists y \text{ productOf}(x,y) \land \text{Tumor}(y)) \leftrightarrow \lozenge_{L}\text{Tumor}(x)$
7. $\forall x \ \square_{L} \text{Tissue}(x) \rightarrow \square_{H} \text{Tissue}(x)$

8. $\lozenge_{S}(\text{Patient}(p1) \land \text{HasPart}(p1,a) \land \text{Tumor}(a) \land \text{Object}(a))$

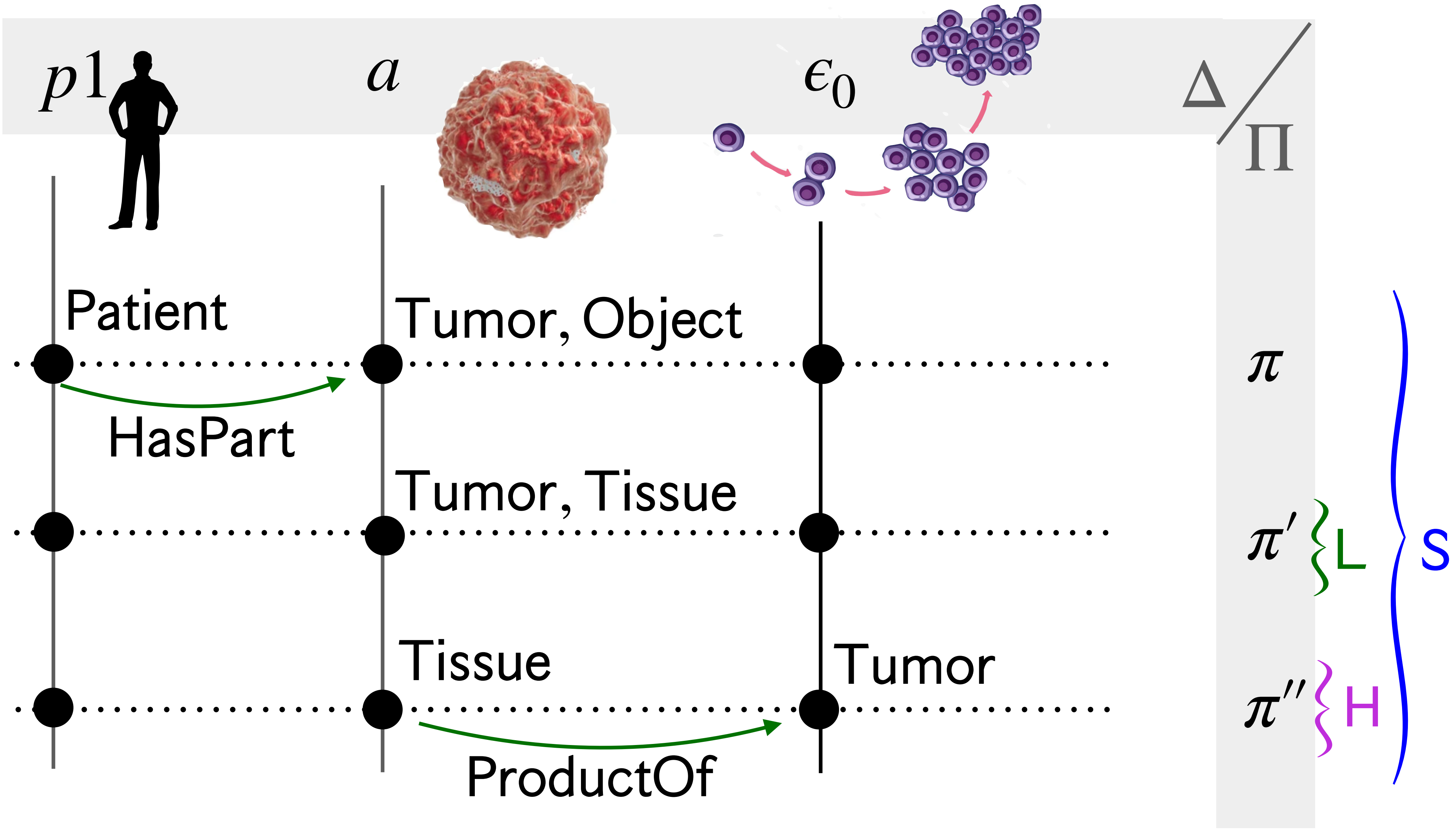
Inferences:

9. $\square_{L}\text{Tumor}(a)$ (5,8)

10. $\square_{L}\text{Tissue}(a)$ (9,3)

11. $\square_{H}\text{Tissue}(a)$ (10,7)

12. $\square_{H} \exists y \text{ productOf}(a,y) \land \text{Tumor}(y)$ (11,6)



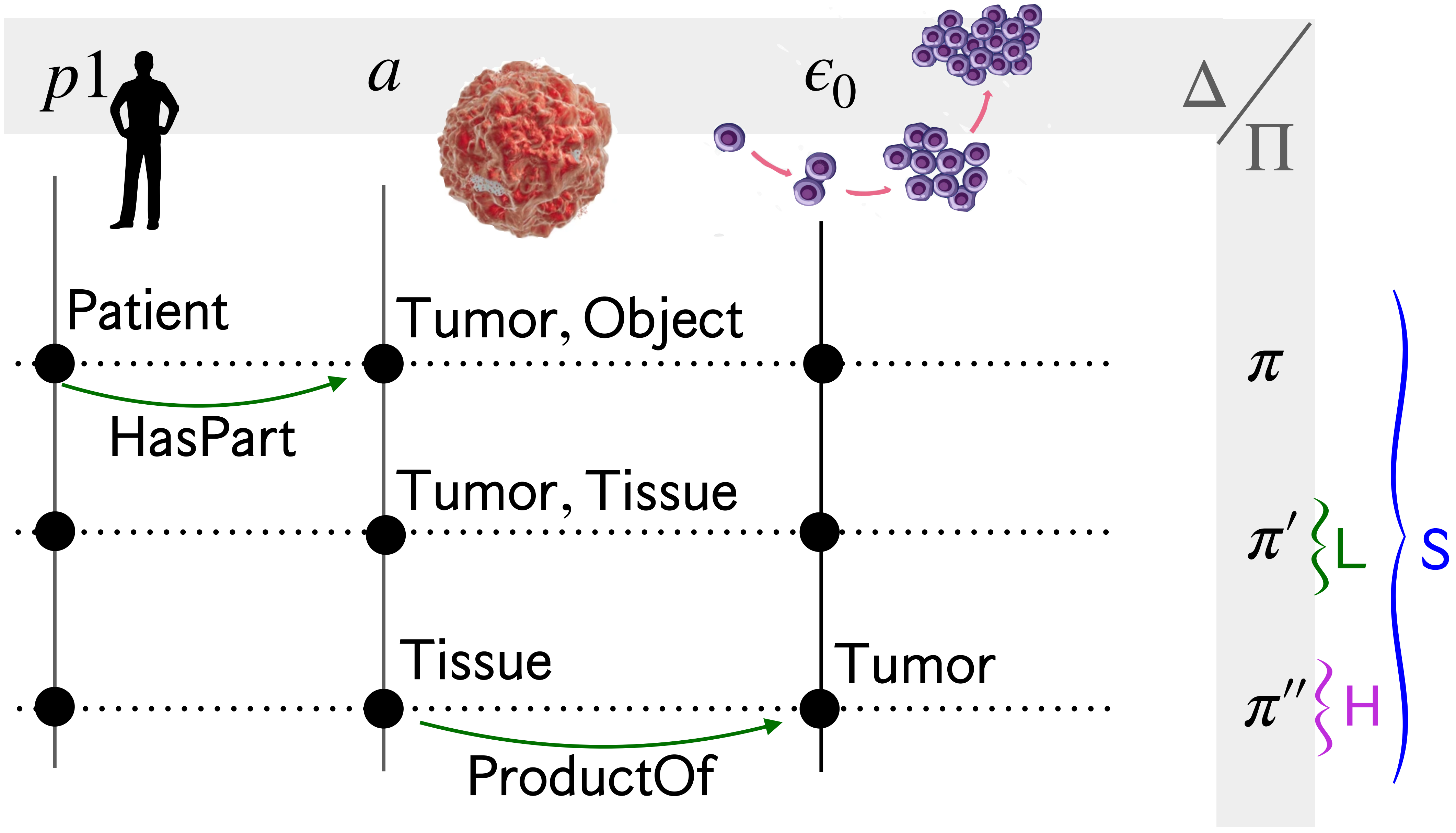
Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \mathsf{Process}(x) \land \mathsf{Tissue}(x))$
2. $\Box_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\Box_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S}\ \land\ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \Diamond_S(\mathsf{Tumor}(x) \land \mathsf{Object}(x)) \rightarrow \Box_L\, \mathsf{Tumor}(x)$
6. $\forall x\ \Box_H\,(\exists y\ \mathsf{productOf}(x,y) \land \mathsf{Tumor}(y)) \leftrightarrow \Diamond_L \mathsf{Tumor}(x)$
7. $\forall x\ \Box_L\, \mathsf{Tissue}(x) \rightarrow \Box_H\, \mathsf{Tissue}(x)$

8. $\Diamond_S(\mathsf{Patient}(p1) \land \mathsf{HasPart}(p1,a) \land \mathsf{Tumor}(a) \land \mathsf{Object}(a))$

Inferences:

9. $\Box_L \mathsf{Tumor}(a)$ (5,8)
10. $\Box_L \mathsf{Tissue}(a)$ (9,3)
11. $\Box_H \mathsf{Tissue}(a)$ (10,7)
12. $\Box_H\, \exists y\ \mathsf{productOf}(a,y) \land \mathsf{Tumor}(y)$ (11,6)
13. $\Box_H\, \exists y\ \mathsf{Process}(y)$ (12,2)



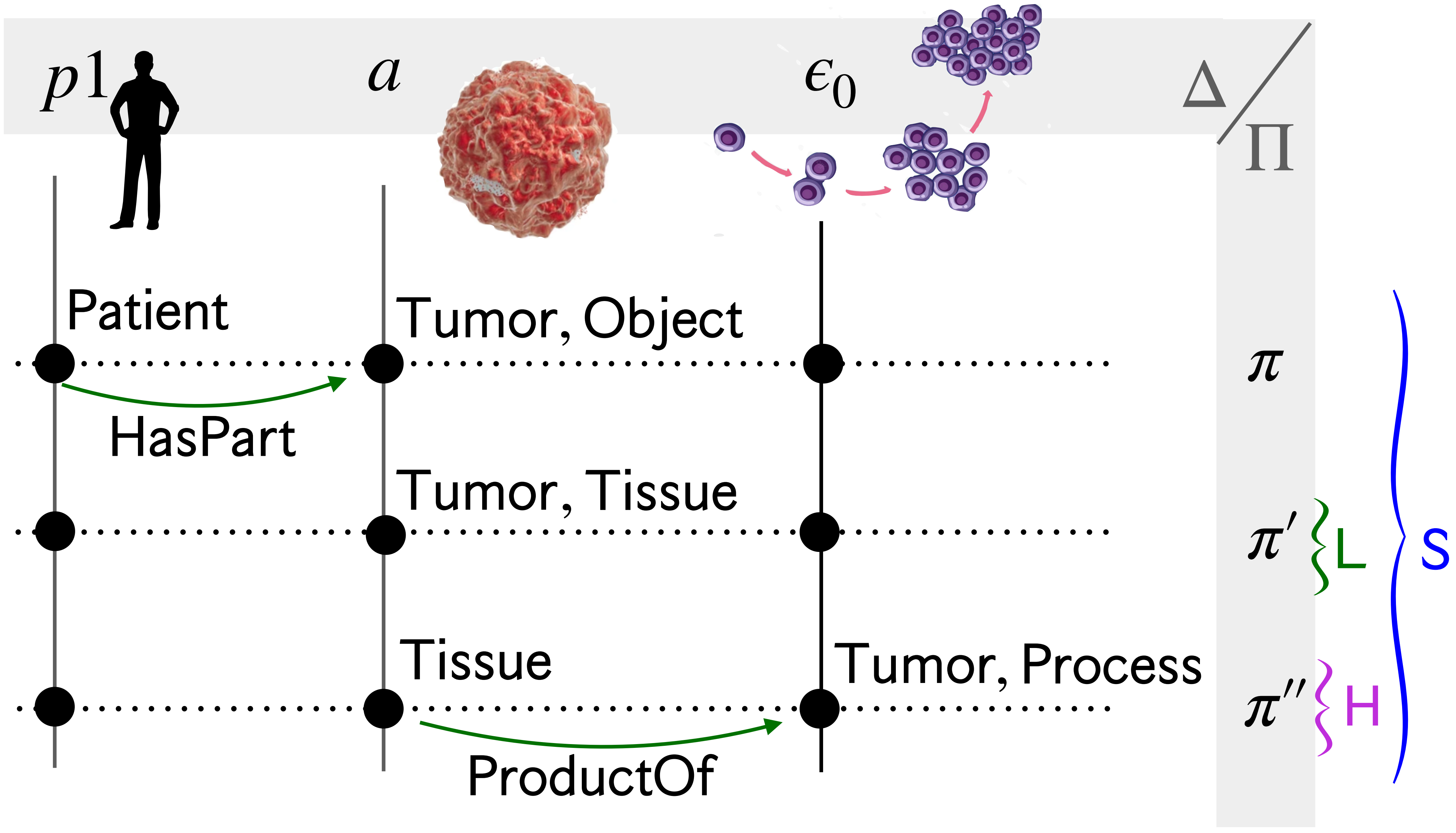
Knowledge Integration - Standpoint Logic

1. $\Box_S(\neg\exists x\ \mathsf{Process}(x) \wedge \mathsf{Tissue}(x))$
2. $\Box_H(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Process}(x))$
3. $\Box_L(\forall x\ \mathsf{Tumor}(x) \rightarrow \mathsf{Tissue}(x))$
4. $\mathsf{L} \preceq \mathsf{S} \ \wedge \ \mathsf{H} \preceq \mathsf{S}$
5. $\forall x\ \Diamond_S(\mathsf{Tumor}(x) \wedge \mathsf{Object}(x)) \rightarrow \Box_L\ \mathsf{Tumor}(x)$
6. $\forall x\ \Box_H(\exists y\ \mathsf{productOf}(x, y) \wedge \mathsf{Tumor}(y)) \leftrightarrow \Diamond_L \mathsf{Tumor}(x)$
7. $\forall x\ \Box_L\ \mathsf{Tissue}(x) \rightarrow \Box_H\ \mathsf{Tissue}(x)$

8. $\Diamond_S(\mathsf{Patient}(p1) \wedge \mathsf{HasPart}(p1,a) \wedge \mathsf{Tumor}(a) \wedge \mathsf{Object}(a))$

Inferences:

9. $\Box_L \mathsf{Tumor}(a)$ (5,8)
10. $\Box_L \mathsf{Tissue}(a)$ (9,3)
11. $\Box_H \mathsf{Tissue}(a)$ (10,7)
12. $\Box_H\ \exists y\ \mathsf{productOf}(a, y) \wedge \mathsf{Tumor}(y)$ (11,6)
13. $\Box_H\ \exists y\ \mathsf{Process}(y)$ (12,2)



Decidability and Complexity

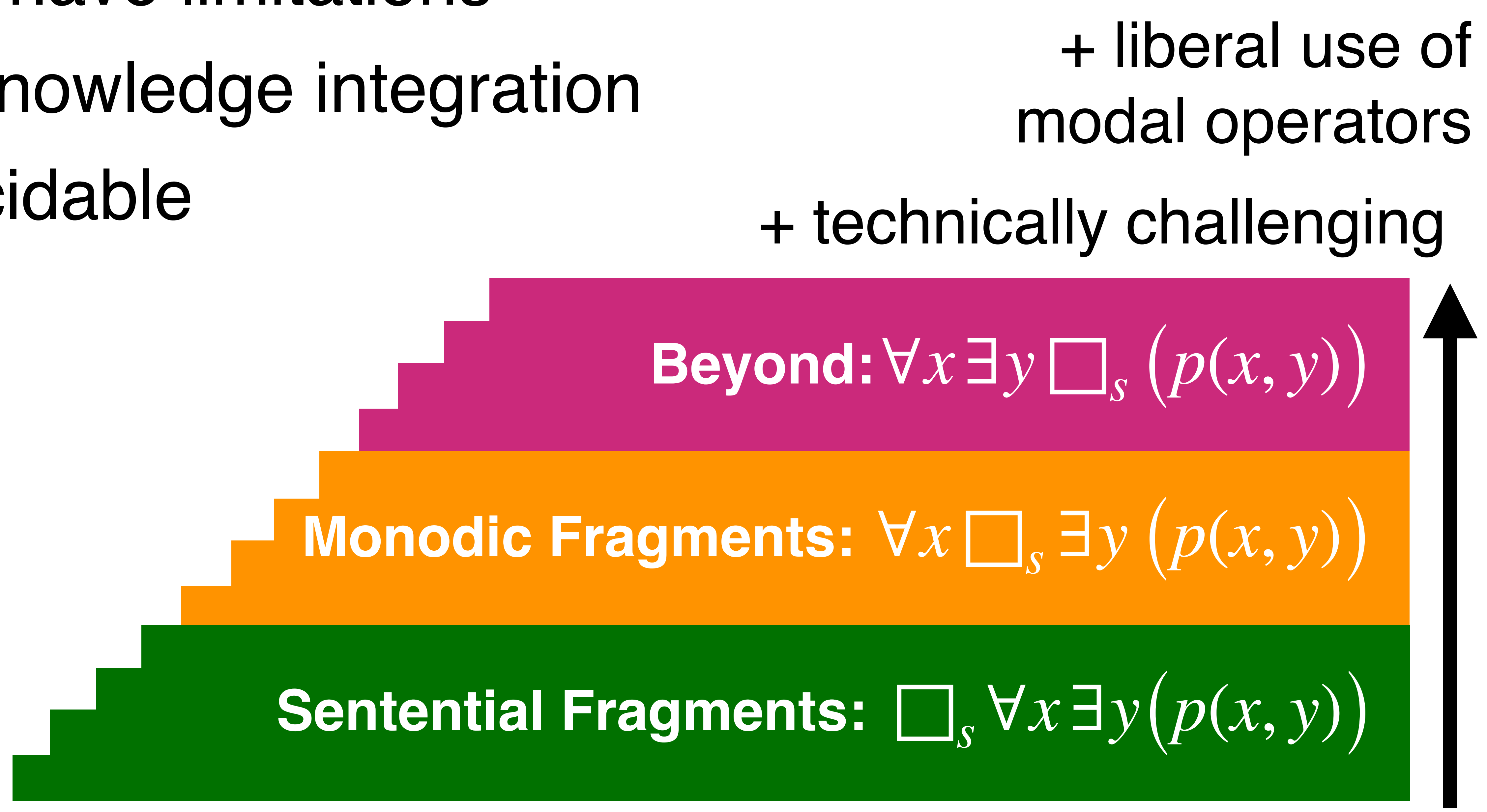
Decidability and Complexity

Goal: Understanding computational cost of reasoning with standpoint-KR languages

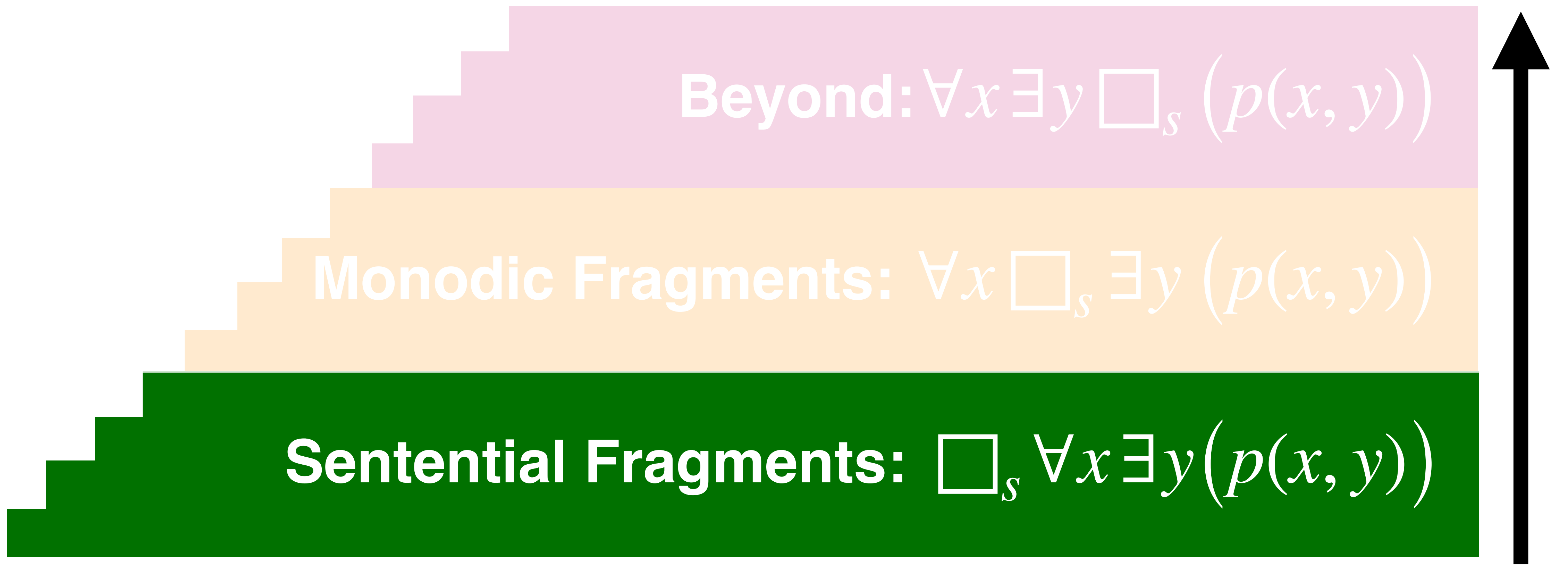
We consider fragments of FOSL.

Important distinction: how much do standpoint operators and quantifiers interleave?

- ⋆ Sentential fragments often preserve complexity, but have limitations
- ⋆ Monodic fragments have important applications in knowledge integration
- ⋆ Beyond monodic modal logics easily become undecidable



Sentential Fragments



Sentential Fragments

Sentential Fragments

Sentential First Order Standpoint Logic (FOSL):

Sentential Fragments

Sentential First Order Standpoint Logic (FOSL):

Definition (Sentential formula):
Let ϕ be a formula of FOSL. We say that ϕ is *sentential* iff for all subformulas of ϕ of the form $\square_e \psi$, all variables occurring in ψ are bound by a quantifier.

Sentential Fragments

Sentential First Order Standpoint Logic (FOSL):

Definition (Sentential formula):
Let ϕ be a formula of FOSL. We say that ϕ is *sentential* iff for all subformulas of ϕ of the form $\square_e \psi$, all variables occurring in ψ are bound by a quantifier.

Eg.:

Sentential Fragments

Sentential First Order Standpoint Logic (FOSL):

Definition (Sentential formula):
Let ϕ be a formula of FOSL. We say that ϕ is *sentential* iff for all subformulas of ϕ of the form $\square_e \psi$, all variables occurring in ψ are bound by a quantifier.

Eg.: 1. $\square_S$[∀x Tumor(x)→(Process(x) ∨ Tissue(x))] ✓

Sentential Fragments

Sentential First Order Standpoint Logic (FOSL):

Definition (Sentential formula):
Let ϕ be a formula of FOSL. We say that ϕ is *sentential* iff for all subformulas of ϕ of the form $\square_e \psi$, all variables occurring in ψ are bound by a quantifier.

Eg.:

1. $\square_S[\forall x\ Tumor(x) \rightarrow (Process(x) \lor Tissue(x))]$ ✓
2. $\Diamond_T[\top] \land \Diamond_P[\top]$ ✓

Sentential Fragments

Sentential First Order Standpoint Logic (FOSL):

Definition (Sentential formula):
Let ϕ be a formula of FOSL. We say that ϕ is *sentential* iff for all subformulas of ϕ of the form $\Box_e \psi$, all variables occurring in ψ are bound by a quantifier.

Eg.:

1. $\Box_S[\forall x \ \mathrm{Tumor}(x) \rightarrow (\mathrm{Process}(x) \vee \mathrm{Tissue}(x))]$
2. $\Diamond_T[T] \wedge \Diamond_P[T]$
3. $\forall x \Box_P[\mathrm{Tissue}(x)] \rightarrow \Box_T[\mathrm{Tissue}(x)]$

Sentential Fragments

Sentential First Order Standpoint Logic (FOSL):

Definition (Sentential formula):
Let ϕ be a formula of FOSL. We say that ϕ is *sentential* iff for all subformulas of ϕ of the form $\Box_e \psi$, all variables occurring in ψ are bound by a quantifier.

Eg.:

1. $\Box_S[\forall x \ \mathtt{Tumor}(x) \rightarrow (\mathtt{Process}(x) \vee \mathtt{Tissue}(x))]$
2. $\Diamond_T[\top] \wedge \Diamond_P[\top]$
3. $\forall x \Box_P[\mathtt{Tissue}(x)] \rightarrow \Box_T[\mathtt{Tissue}(x)]$

Theorem (Small Model Property):
A sentential FOSL formula ϕ is satisfiable iff it has a model with at most ϕ precisifications. That is, for sentential FOSL, satisfiability and $|\phi|$-satisfiability coincide.

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \rightarrow \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \rightarrow \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Example: $\phi = \neg \Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R} \cap \mathsf{B}}\, \mathtt{p}(a)$

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \to \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Example: $\phi = \neg\Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R}\cap\mathsf{B}}\, \mathtt{p}(a)$

$$\begin{aligned}
\mathrm{Trans}_3(\phi) = \bigwedge_{\pi \in \Pi_3} \\
\neg((\mathsf{R}_1 \to \mathtt{p}_1(a)) \wedge (\mathsf{R}_2 \to \mathtt{p}_2(a)) \wedge (\mathsf{R}_3 \to \mathtt{p}_3(a))) \\
\wedge \\
(((\mathsf{R}_1 \wedge \mathsf{B}_1) \to \mathtt{p}_1(a)) \wedge ((\mathsf{R}_2 \wedge \mathsf{B}_2) \to \mathtt{p}_2(a)) \wedge ((\mathsf{R}_3 \wedge \mathsf{B}_3) \to \mathtt{p}_3(a))) \\
\wedge \\
*_1 \wedge *_2 \wedge *_3,
\end{aligned}$$

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi\in\Pi_n} \mathrm{trans}_n(\pi,\phi) \wedge \bigwedge_{\pi\in\Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi,\mathtt{P}(t_1,\ldots,t_k)) &= \mathtt{P}_\pi(t_1,\ldots,t_k)\\
\mathrm{trans}_n(\pi,\neg\psi) &= \neg\mathrm{trans}_n(\pi,\psi)\\
\mathrm{trans}_n(\pi,\psi_1\wedge\psi_2) &= \mathrm{trans}_n(\pi,\psi_1)\wedge\mathrm{trans}_n(\pi,\psi_2)\\
\mathrm{trans}_n(\pi,\forall x\psi) &= \forall x(\mathrm{trans}_n(\pi,\psi))\\
\mathrm{trans}_n(\pi',\square_{\mathsf{e}}\,\psi) &= \bigwedge_{\pi\in\Pi_n}(\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e})\rightarrow\mathrm{trans}_n(\pi,\psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{s}) &= \mathsf{s}_\pi\\
\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_1\cup\mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_1)\vee\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_2)\\
\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_1\cap\mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_1)\wedge\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_2)\\
\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_1\setminus\mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_1)\wedge\neg\mathrm{trans}_{\mathcal{E}}(\pi,\mathsf{e}_2)
\end{aligned}$$

Example: $\phi = \neg\square_{\mathsf{R}}\,\mathtt{p}(a) \wedge \square_{\mathsf{R}\cap\mathsf{B}}\,\mathtt{p}(a)$

$$\begin{aligned}
\mathrm{Trans}_3(\phi) = \bigwedge_{\pi\in\Pi_3}&\\
\neg((\mathsf{R}_1\rightarrow\mathtt{p}_1(a))\wedge(\mathsf{R}_2\rightarrow\mathtt{p}_2(a))\wedge(\mathsf{R}_3\rightarrow\mathtt{p}_3(a)))&\\
\wedge&\\
(((\mathsf{R}_1\wedge\mathsf{B}_1)\rightarrow\mathtt{p}_1(a))\wedge((\mathsf{R}_2\wedge\mathsf{B}_2)\rightarrow\mathtt{p}_2(a))\wedge((\mathsf{R}_3\wedge\mathsf{B}_3)\rightarrow\mathtt{p}_3(a)))&\\
\wedge&\\
*_1\wedge *_2\wedge *_3,&
\end{aligned}$$

FOL model

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

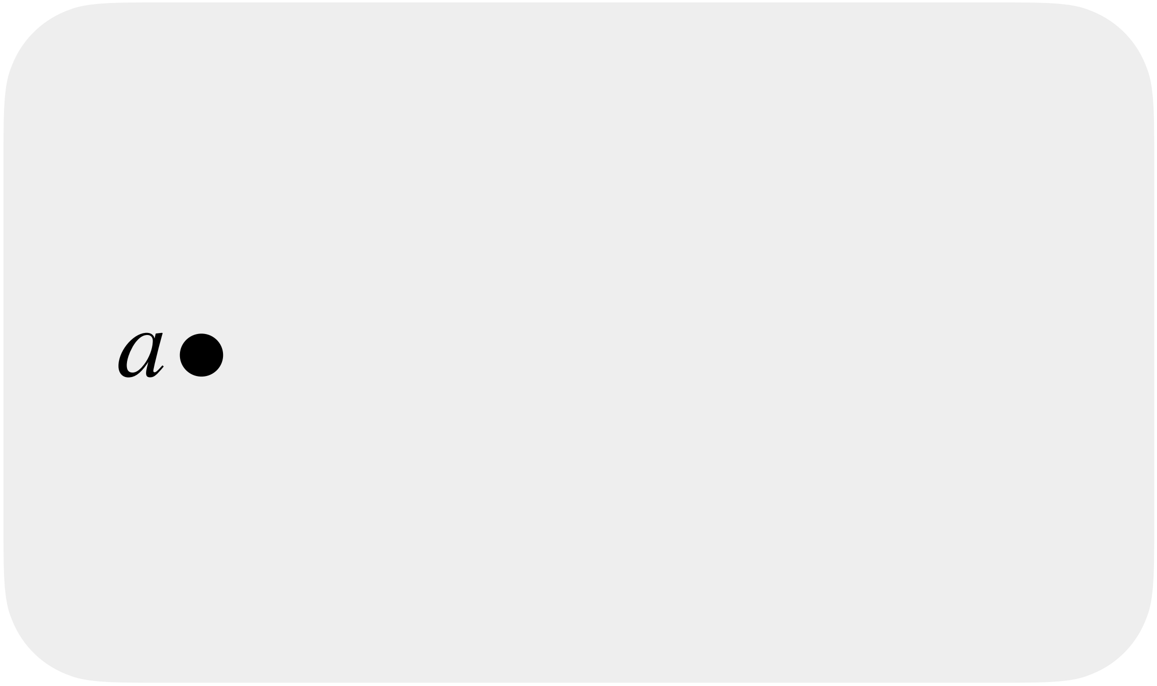
$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \to \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Example: $\phi = \neg \Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R}\cap\mathsf{B}}\, \mathtt{p}(a)$

$$\begin{aligned}
&\mathrm{Trans}_3(\phi) = \textstyle\bigwedge_{\pi \in \Pi_3} \\
&\neg((\mathsf{R}_1 \to \mathtt{p}_1(a)) \wedge (\mathsf{R}_2 \to \mathtt{p}_2(a)) \wedge (\mathsf{R}_3 \to \mathtt{p}_3(a))) \\
&\wedge \\
&(((\mathsf{R}_1 \wedge \mathsf{B}_1) \to \mathtt{p}_1(a)) \wedge ((\mathsf{R}_2 \wedge \mathsf{B}_2) \to \mathtt{p}_2(a)) \wedge ((\mathsf{R}_3 \wedge \mathsf{B}_3) \to \mathtt{p}_3(a))) \\
&\wedge \\
&*_1 \wedge *_2 \wedge *_3,
\end{aligned}$$



FOL model

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \rightarrow \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Example: $\phi = \neg \Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R} \cap \mathsf{B}}\, \mathtt{p}(a)$

$$\begin{aligned}
\mathrm{Trans}_3(\phi) = \bigwedge_{\pi \in \Pi_3} \\
\neg((\mathsf{R}_1 \rightarrow \mathtt{p}_1(a)) \wedge (\mathsf{R}_2 \rightarrow \mathtt{p}_2(a)) \wedge (\mathsf{R}_3 \rightarrow \mathtt{p}_3(a))) \\
\wedge \\
(((\mathsf{R}_1 \wedge \mathsf{B}_1) \rightarrow \mathtt{p}_1(a)) \wedge ((\mathsf{R}_2 \wedge \mathsf{B}_2) \rightarrow \mathtt{p}_2(a)) \wedge ((\mathsf{R}_3 \wedge \mathsf{B}_3) \rightarrow \mathtt{p}_3(a))) \\
\wedge \\
*_1 \wedge *_2 \wedge *_3,
\end{aligned}$$

	$\neg p_1$	$*_1$	R_1	$\neg\mathsf{B}_1$
a ●	p_2	$*_2$	R_2	B_2
	p_3	$*_3$	$\neg\mathsf{R}_3$	B_3

FOL model

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

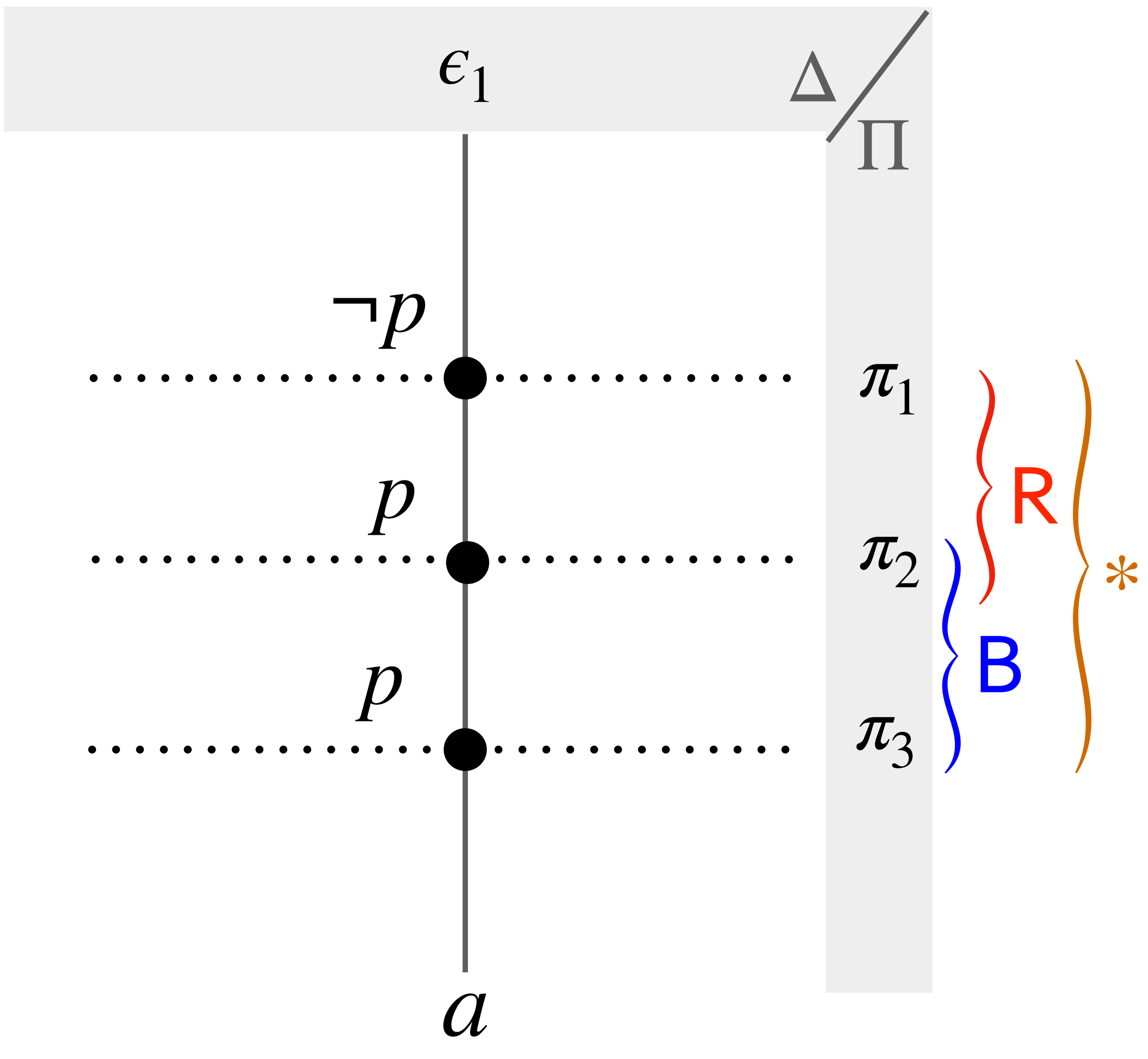
$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \rightarrow \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

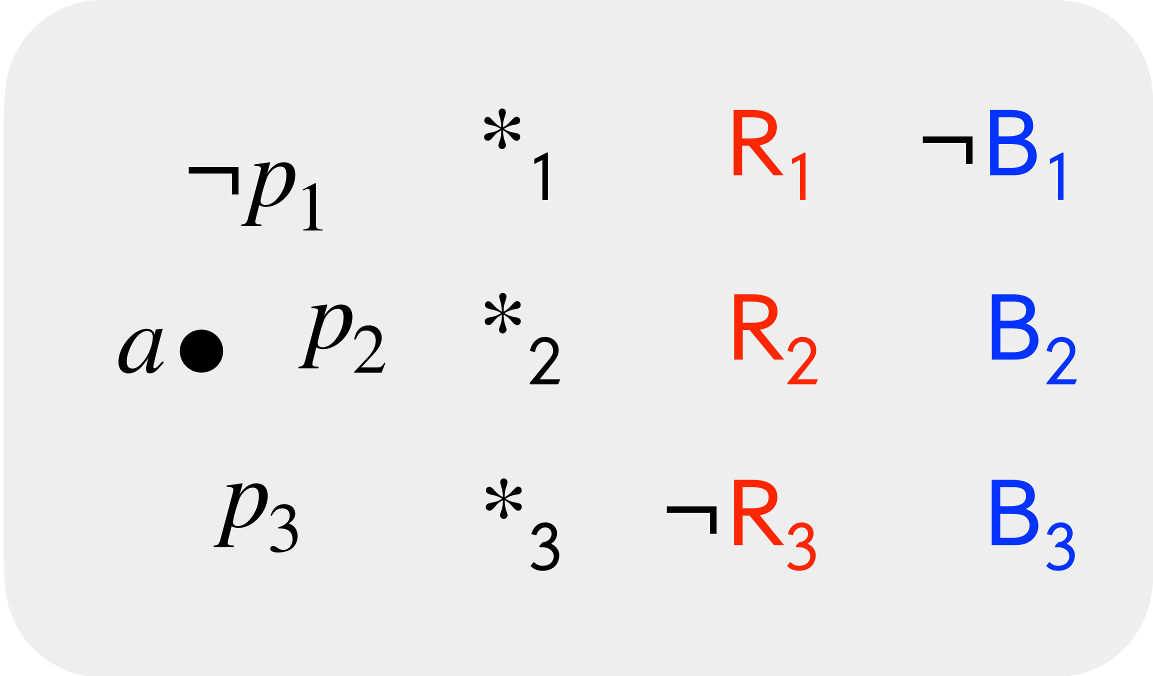
$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Example: $\phi = \neg\Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R}\cap\mathsf{B}}\, \mathtt{p}(a)$

$$\begin{aligned}
\mathrm{Trans}_3(\phi) = \textstyle\bigwedge_{\pi \in \Pi_3} \\
\neg((\mathsf{R}_1 \rightarrow \mathtt{p}_1(a)) \wedge (\mathsf{R}_2 \rightarrow \mathtt{p}_2(a)) \wedge (\mathsf{R}_3 \rightarrow \mathtt{p}_3(a))) \\
\wedge \\
(((\mathsf{R}_1 \wedge \mathsf{B}_1) \rightarrow \mathtt{p}_1(a)) \wedge ((\mathsf{R}_2 \wedge \mathsf{B}_2) \rightarrow \mathtt{p}_2(a)) \wedge ((\mathsf{R}_3 \wedge \mathsf{B}_3) \rightarrow \mathtt{p}_3(a))) \\
\wedge \\
*_1 \wedge *_2 \wedge *_3,
\end{aligned}$$



FOSL model



FOL model

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) = \mathtt{P}_\pi(t_1, \ldots, t_k)$$

$$\mathrm{trans}_n(\pi, \neg\psi) = \neg\mathrm{trans}_n(\pi, \psi)$$

$$\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) = \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2)$$

$$\mathrm{trans}_n(\pi, \forall x \psi) = \forall x(\mathrm{trans}_n(\pi, \psi))$$

$$\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) = \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \rightarrow \mathrm{trans}_n(\pi, \psi))$$

$$\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) = \mathsf{s}_\pi$$

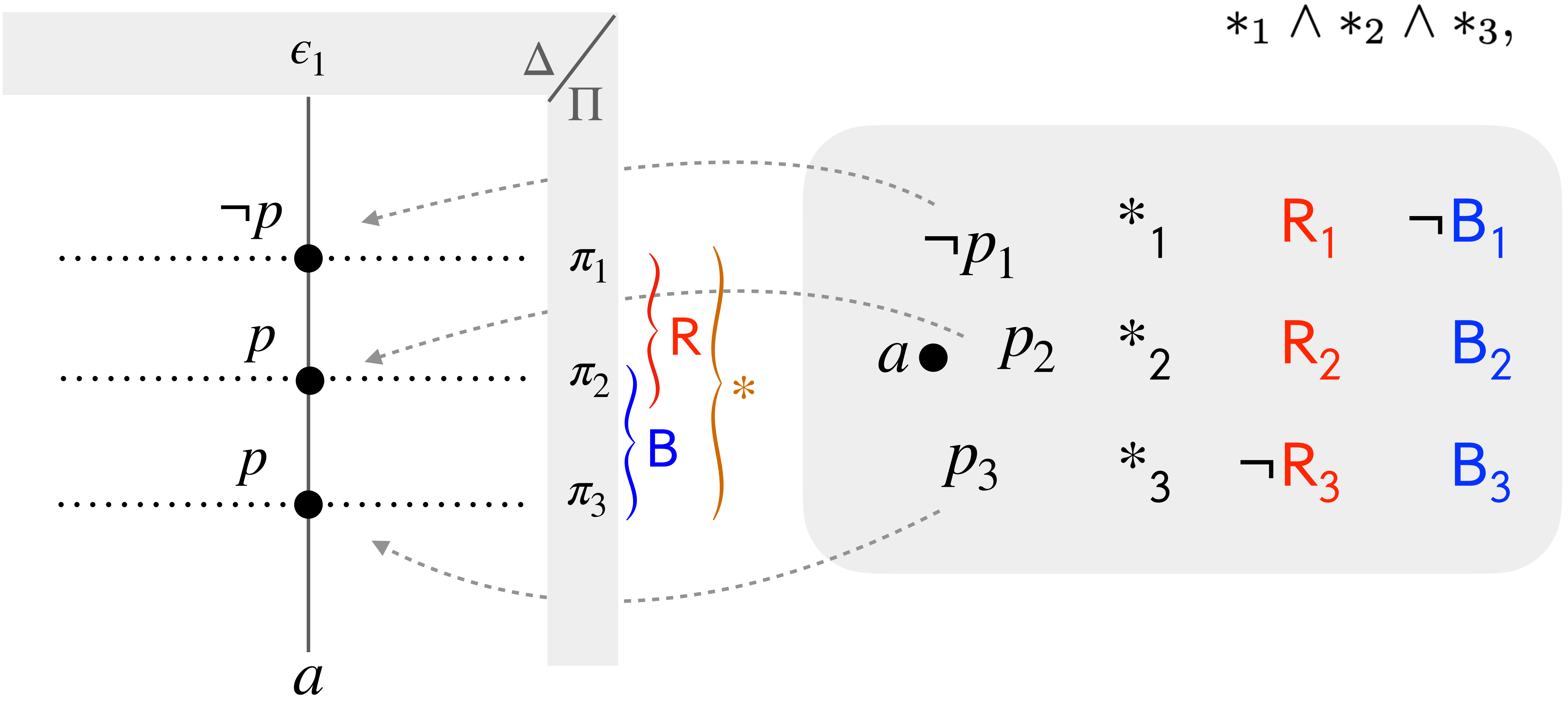
$$\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) = \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)$$

$$\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) = \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)$$

$$\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) = \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)$$

Example: $\phi = \neg\Box_{\mathsf{R}}\, \mathtt{p}(a) \wedge \Box_{\mathsf{R}\cap\mathsf{B}}\, \mathtt{p}(a)$

$$\begin{aligned}
\mathrm{Trans}_3(\phi) = {} & \textstyle\bigwedge_{\pi \in \Pi_3} \\
& \neg((\mathsf{R}_1 \rightarrow \mathtt{p}_1(a)) \wedge (\mathsf{R}_2 \rightarrow \mathtt{p}_2(a)) \wedge (\mathsf{R}_3 \rightarrow \mathtt{p}_3(a))) \\
& \wedge \\
& (((\mathsf{R}_1 \wedge \mathsf{B}_1) \rightarrow \mathtt{p}_1(a)) \wedge ((\mathsf{R}_2 \wedge \mathsf{B}_2) \rightarrow \mathtt{p}_2(a)) \wedge ((\mathsf{R}_3 \wedge \mathsf{B}_3) \rightarrow \mathtt{p}_3(a))) \\
& \wedge \\
& *_1 \wedge *_2 \wedge *_3,
\end{aligned}$$



FOSL model **FOL model**

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \rightarrow \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \to \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Lemma:
A formula ϕ is n-satisfiable in FOSL if and only if $\mathrm{Trans}_n(\phi)$ is satisfiable in first-order logic.

Sentential Fragments

(n-)Equisatisfiable Translation to Plain FOL

$$\mathrm{Trans}_n(\phi) = \bigwedge_{\pi \in \Pi_n} \mathrm{trans}_n(\pi, \phi) \wedge \bigwedge_{\pi \in \Pi_n} *_\pi,$$

$$\begin{aligned}
\mathrm{trans}_n(\pi, \mathtt{P}(t_1, \ldots, t_k)) &= \mathtt{P}_\pi(t_1, \ldots, t_k) \\
\mathrm{trans}_n(\pi, \neg\psi) &= \neg\mathrm{trans}_n(\pi, \psi) \\
\mathrm{trans}_n(\pi, \psi_1 \wedge \psi_2) &= \mathrm{trans}_n(\pi, \psi_1) \wedge \mathrm{trans}_n(\pi, \psi_2) \\
\mathrm{trans}_n(\pi, \forall x \psi) &= \forall x(\mathrm{trans}_n(\pi, \psi)) \\
\mathrm{trans}_n(\pi', \Box_{\mathsf{e}}\, \psi) &= \bigwedge_{\pi \in \Pi_n} (\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}) \to \mathrm{trans}_n(\pi, \psi))
\end{aligned}$$

$$\begin{aligned}
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{s}) &= \mathsf{s}_\pi \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cup \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \vee \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \cap \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2) \\
\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1 \setminus \mathsf{e}_2) &= \mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_1) \wedge \neg\mathrm{trans}_{\mathcal{E}}(\pi, \mathsf{e}_2)
\end{aligned}$$

Lemma:

A formula ϕ is n-satisfiable in FOSL if and only if $\mathrm{Trans}_n(\phi)$ is satisfiable in first-order logic.

Theorem:

Let F be a "translation-friendly" fragment of FOL. Then the satisfiability of the sentential standpoint-F fragment of FOSL,

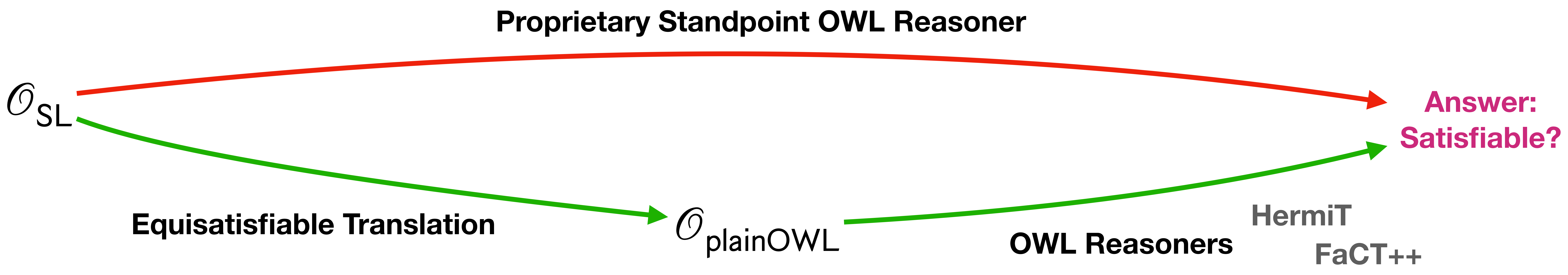
- is decidable iff it is for F, and
- has the same complexity as F (if at least NP)

Decidable Sentential Framgments

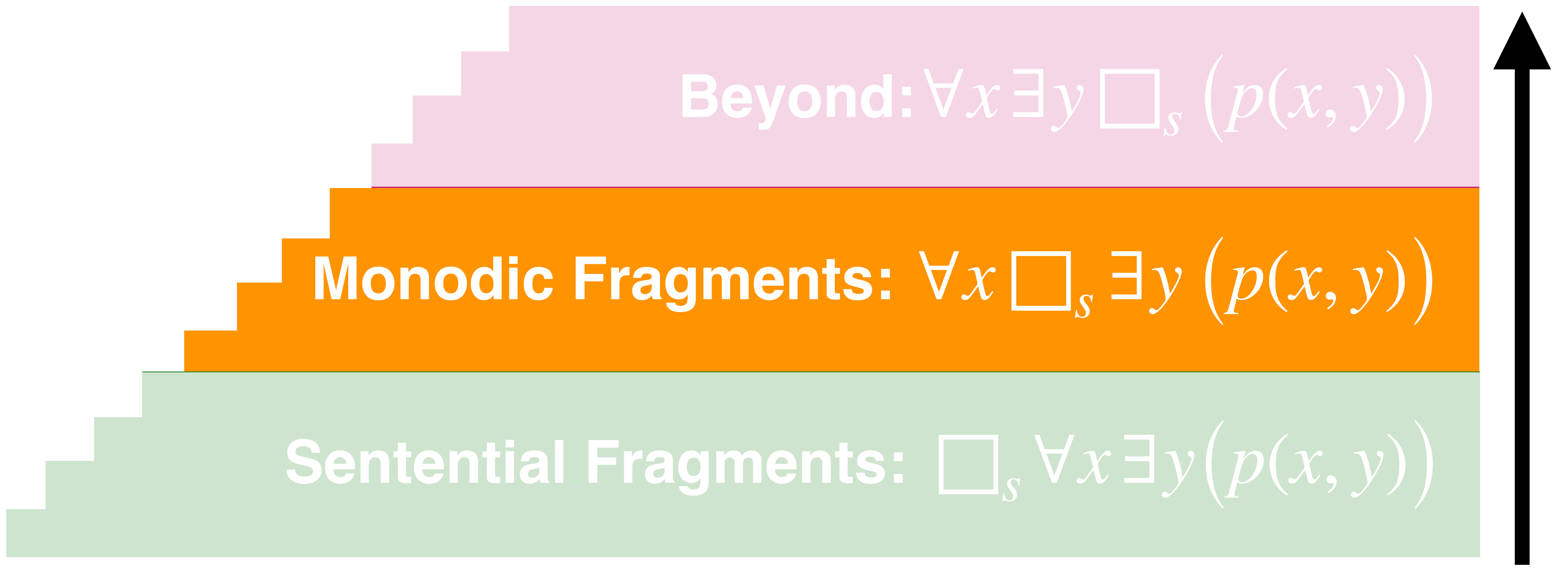
- By small model property and generic translation, complexity of decidable fragments is preserved:
  - ➡ S Guarded fragment (GF) → 2ExpTime
  - ➡ S Triguarded fragment (TGF) → 2NExpTime
  - ➡ S Counting 2-variable fragment (C^2) → NExpTime
  - ➡ Standpoint OWL 2 → 2NExpTime (some extra tricks required)

Decidable Sentential Framgments

- By small model property and generic translation, complexity of decidable fragments is preserved:
  - ➡ S Guarded fragment (GF) → 2ExpTime
  - ➡ S Triguarded fragment (TGF) → 2NExpTime
  - ➡ S Counting 2-variable fragment (C^2) → NExpTime
  - ➡ Standpoint OWL 2 → 2NExpTime (some extra tricks required)
- Result via polynomial equisatisfiable translation → practical implementations



Monodic Fragments



Standpoint $\mathcal{EL}^+$

The description logic $\mathcal{EL}$

The description logic $\mathcal{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

The description logic $\mathcal{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathrm{I}} \rangle$ of concept, role, individual names

Syntax:

The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

Process $\sqcap$ Tissue

The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The description logic $\mathcal{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The **set of axioms** includes:

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

Process $\sqcap$ Tissue

$\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$

The description logic $\mathscr{E}\mathscr{L}$

Vocabulary $\langle N_C, N_R, N_I \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$

With $A \in N_C$, $r \in N_R$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$

The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\quad\quad$ $\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $\quad C \sqsubseteq D$
- Assertions: $C(a)$, $\quad r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $\quad\quad$ $(\exists \mathsf{patientPart} \,.\, \mathsf{Tumor})(p)$

The description logic $\mathscr{E}\mathscr{L}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

Semantics:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$\big(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}\big)$ $\big(\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}\big)(p)$

The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathrm{C}, N_\mathrm{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$

With $A \in N_\mathrm{C}, r \in N_\mathrm{R}$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$\left(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}\right)$ $\left(\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}\right)(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

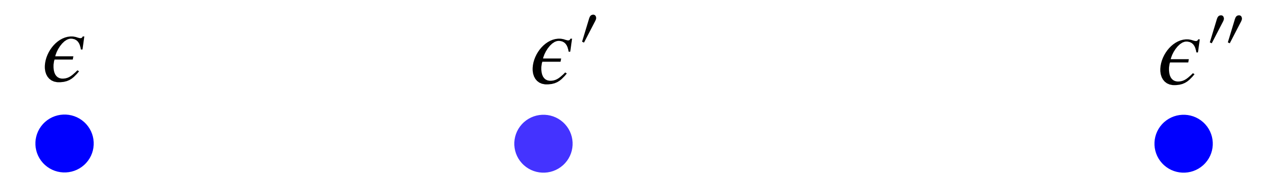
$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists \mathsf{patientPart}\,.\,\mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



The description logic $\mathcal{EL}$

Vocabulary $\langle N_{\mathrm{C}}, N_{\mathrm{R}}, N_{\mathrm{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C$$

With $A \in N_{\mathrm{C}}, r \in N_{\mathrm{R}}$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists\mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists\mathsf{patientPart}\,.\,\mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{EL}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathrm{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$

With $A \in N_{\mathsf{C}}$, $r \in N_{\mathsf{R}}$

Tissue

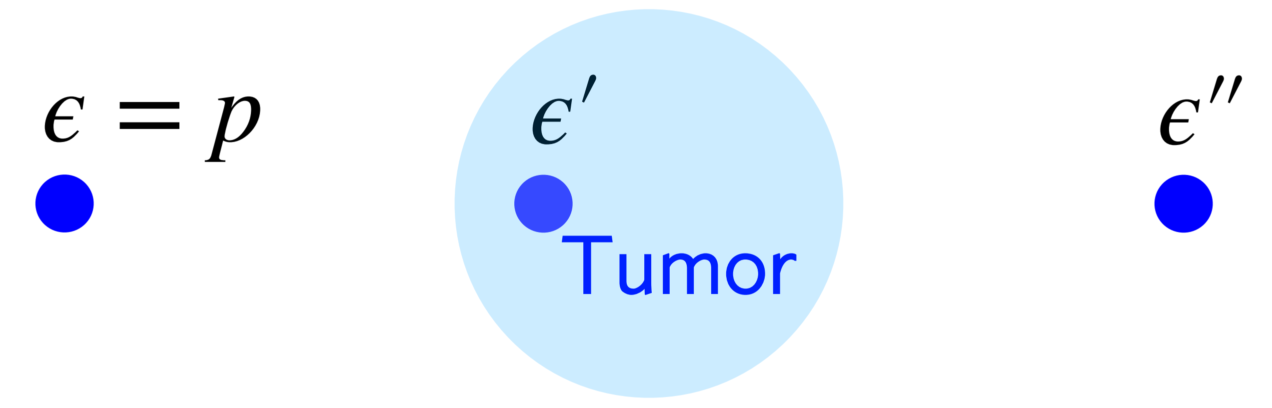
Process $\sqcap$ Tissue $\qquad \exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $\qquad C \sqsubseteq D$
- Assertions: $C(a), \quad r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}) \qquad (\exists \mathsf{patientPart} \,.\, \mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{E}\mathscr{L}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

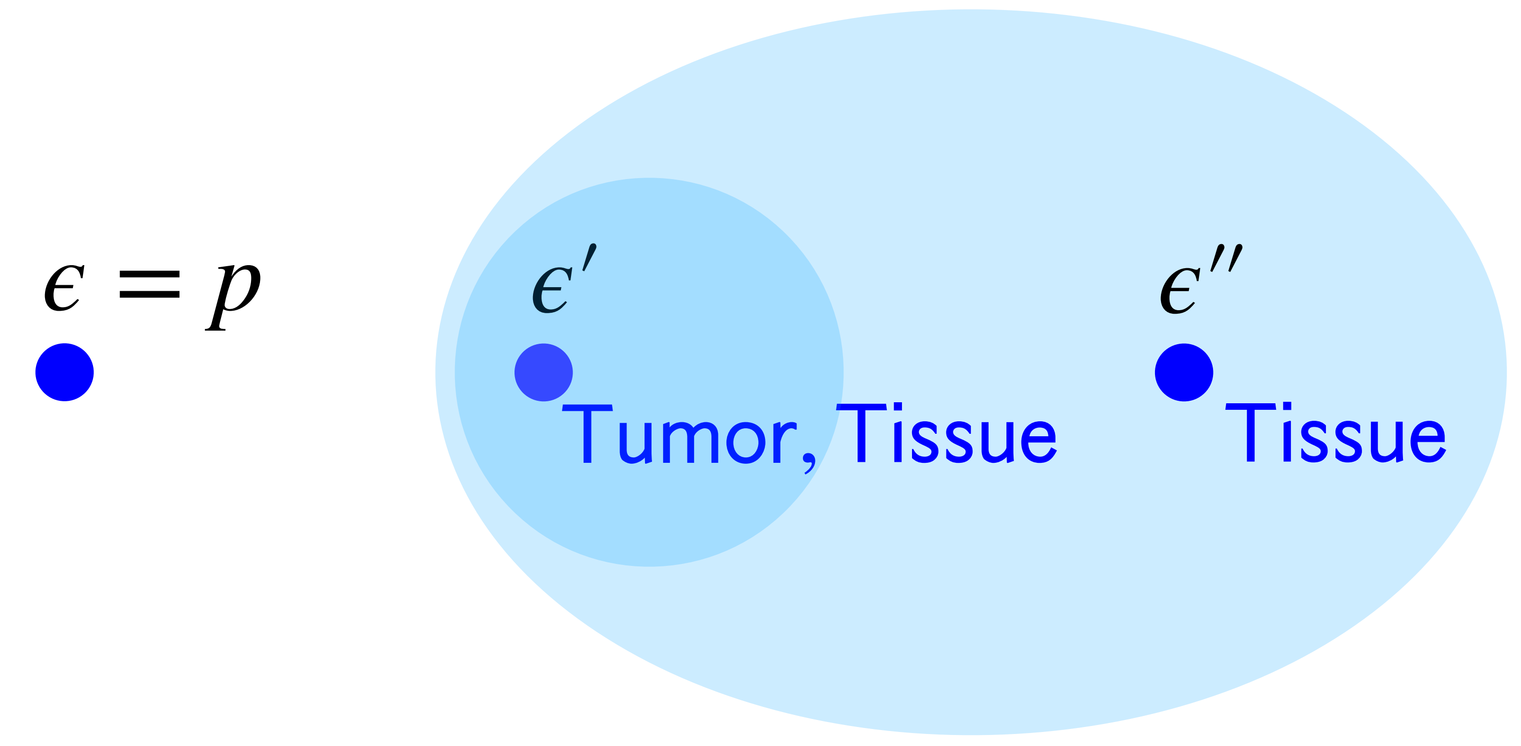
Process $\sqcap$ Tissue $\quad$ $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $\quad C \sqsubseteq D$
- Assertions: $\quad C(a)$, $\quad r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue})$ $\quad$ $(\exists \text{patientPart} \,.\, \text{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{E}\mathscr{L}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$

With $A \in N_{\mathsf{C}}$, $r \in N_{\mathsf{R}}$

Tissue

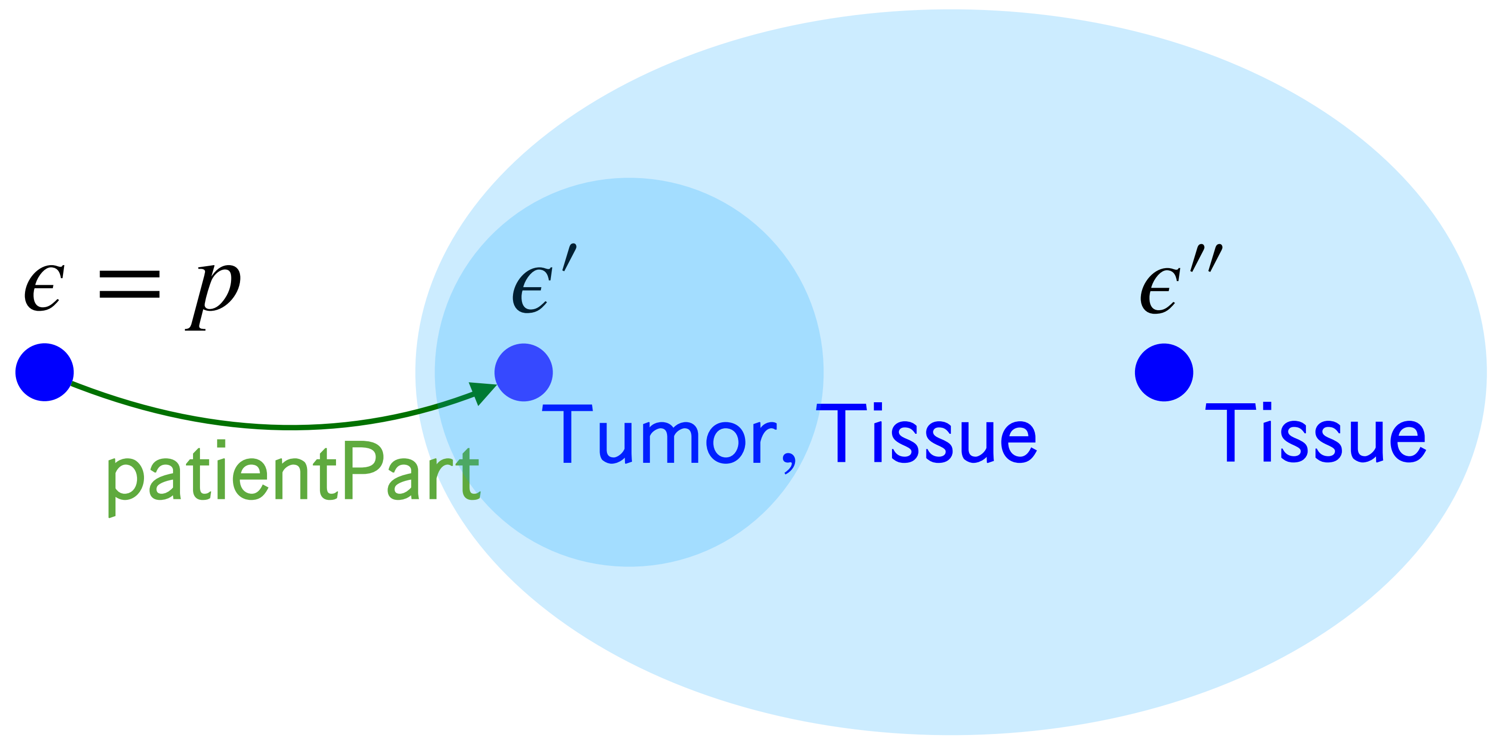
$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$\big(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}\big)$ $\big(\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}\big)(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

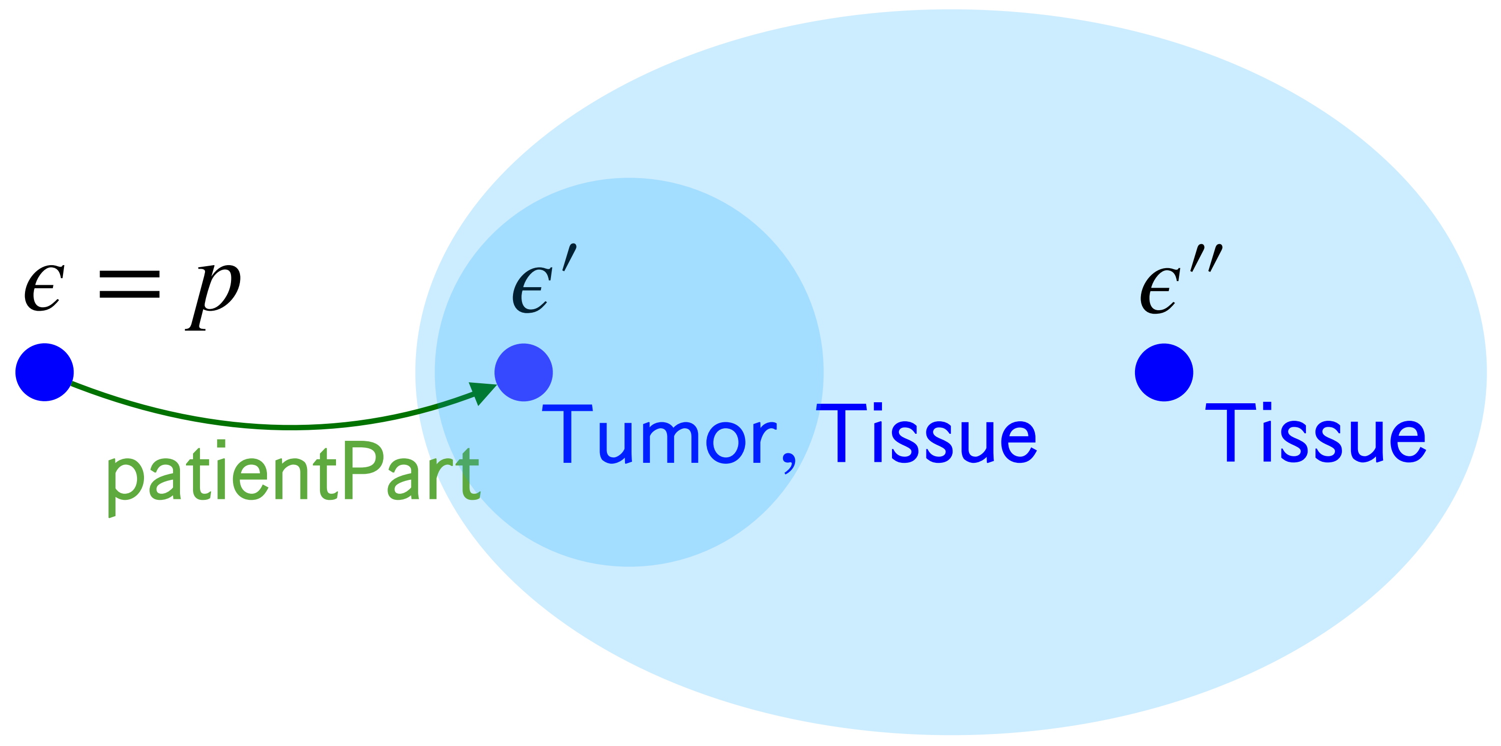
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue})$ $(\exists \text{patientPart} \,.\, \text{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



The description logic $\mathscr{E}\mathscr{L}$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue

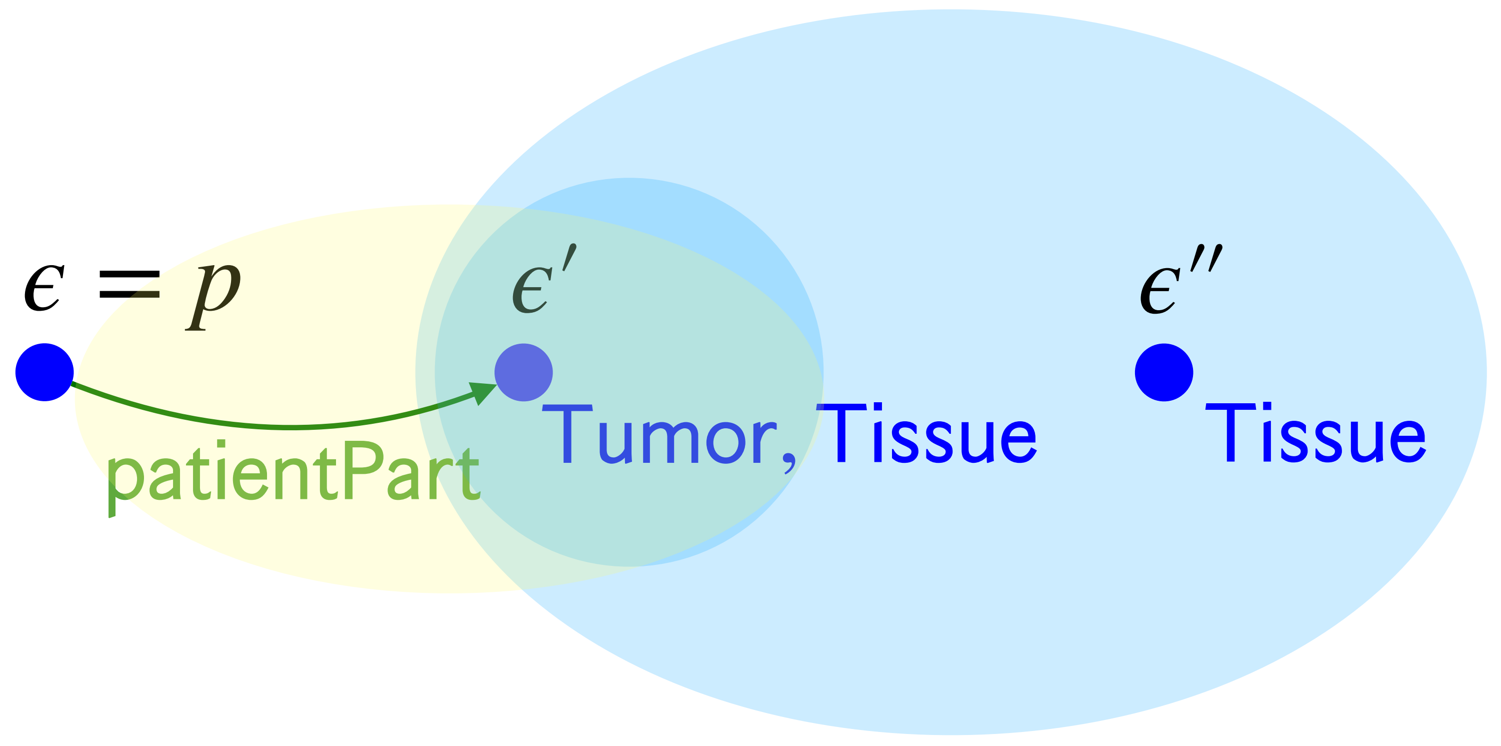
$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\quad$ $\exists \mathsf{patientPart} \,.\, \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $\quad C \sqsubseteq D$
- Assertions: $\quad C(a), \quad r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $\quad$ $(\exists \mathsf{patientPart} \,.\, \mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathcal{EL}$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart} . \mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists \mathsf{patientPart} . \mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$

The description logic $\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C$$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue

Process $\sqcap$ Tissue $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue})$ $(\exists\text{patientPart} \,.\, \text{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$

The description logic $\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C \mid \exists r \,.\, \mathit{Self}$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue

Process $\sqcap$ Tissue $\quad$ $\exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs $\quad C \sqsubseteq D$
- Assertions: $C(a), \quad r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}) \quad (\exists \mathsf{patientPart} \,.\, \mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$

The description logic $\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self}$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue $\exists \mathsf{diagnoses}\,.\,\mathsf{Self}$

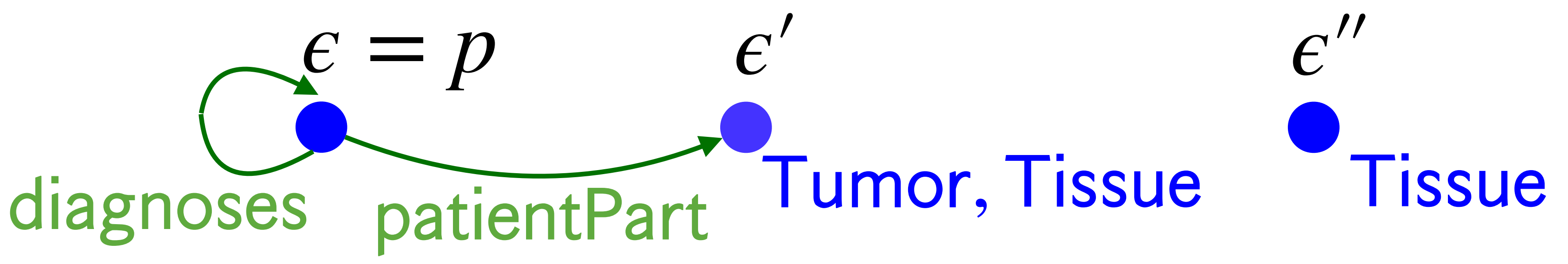
$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs $C \sqsubseteq D$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\exists \mathsf{patientPart}\,.\,\mathsf{Tumor})(p)$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



The description logic $\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathsf{I}} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C \mid \exists r \,.\, \mathit{Self}$$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue $\quad \exists$diagnoses . Self

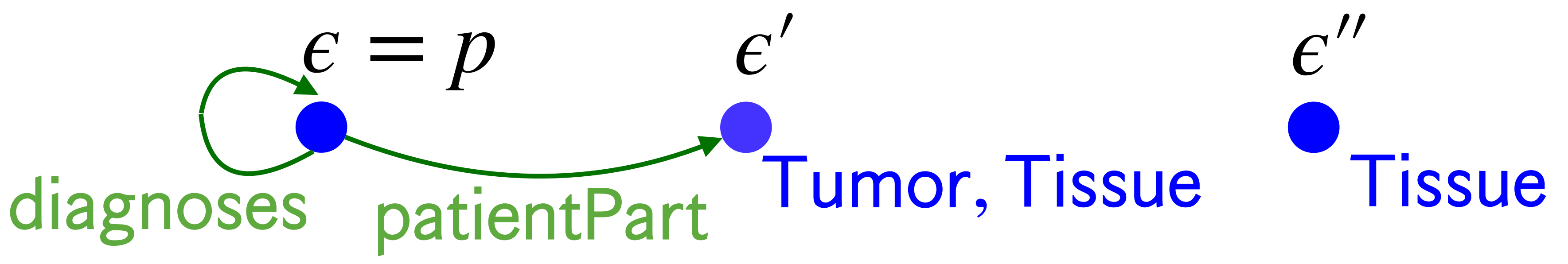
Process $\sqcap$ Tissue $\quad \exists$patientPart . Tumor

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a, b)$

(Tumor $\sqsubseteq$ Tissue)

Semantics: $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$



The description logic $\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C \mid \exists r . \mathit{Self}$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

Tissue $\quad \exists \text{diagnoses} . \text{Self}$

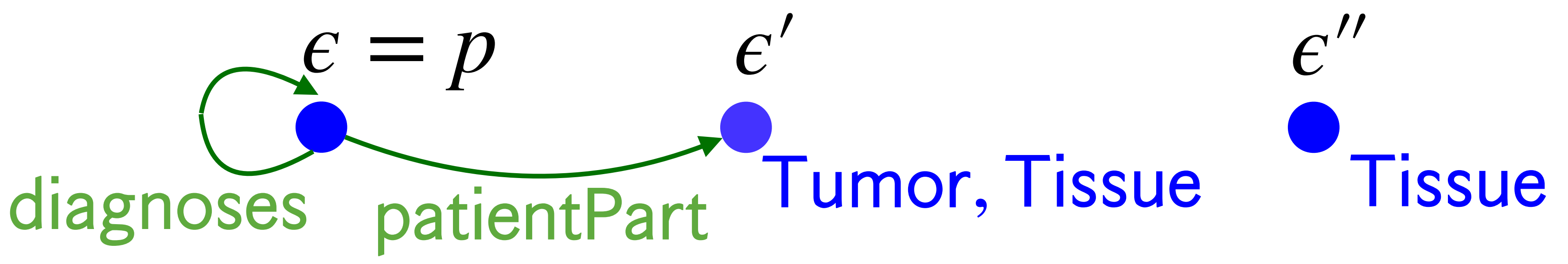
$\text{Process} \sqcap \text{Tissue} \quad \exists \text{patientPart} . \text{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a, b)$

$(\text{Tumor} \sqsubseteq \text{Tissue}) \qquad (\text{patientPart} \circ \text{hasPart} \sqsubseteq \text{patientPart})$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



The description logic $\mathcal{EL}+$

Vocabulary $\langle N_\mathrm{C}, N_\mathrm{R}, N_\mathrm{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r . C \mid \exists r . \mathit{Self}$

With $A \in N_\mathrm{C}$, $r \in N_\mathrm{R}$

Tissue $\quad \exists \mathrm{diagnoses} . \mathrm{Self}$

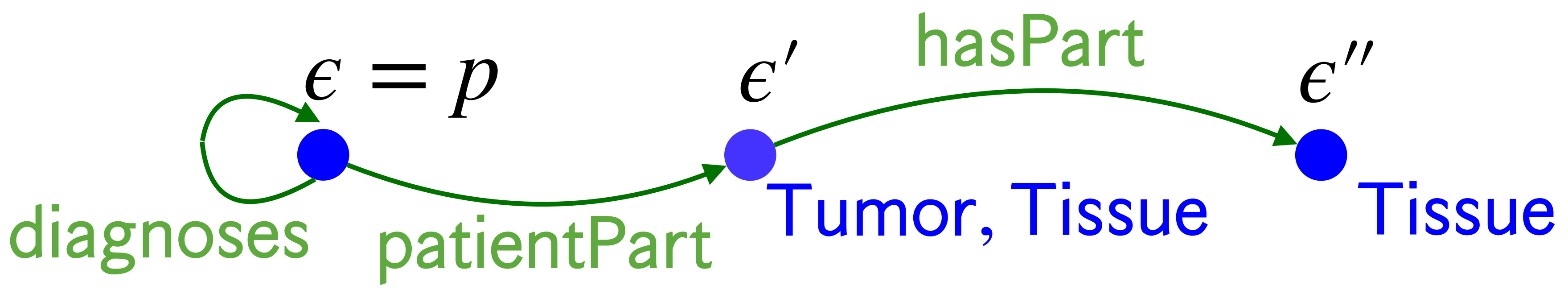
$\mathrm{Process} \sqcap \mathrm{Tissue} \quad \exists \mathrm{patientPart} . \mathrm{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

$(\mathrm{Tumor} \sqsubseteq \mathrm{Tissue}) \quad (\mathrm{patientPart} \circ \mathrm{hasPart} \sqsubseteq \mathrm{patientPart})$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



The description logic $\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self}$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$

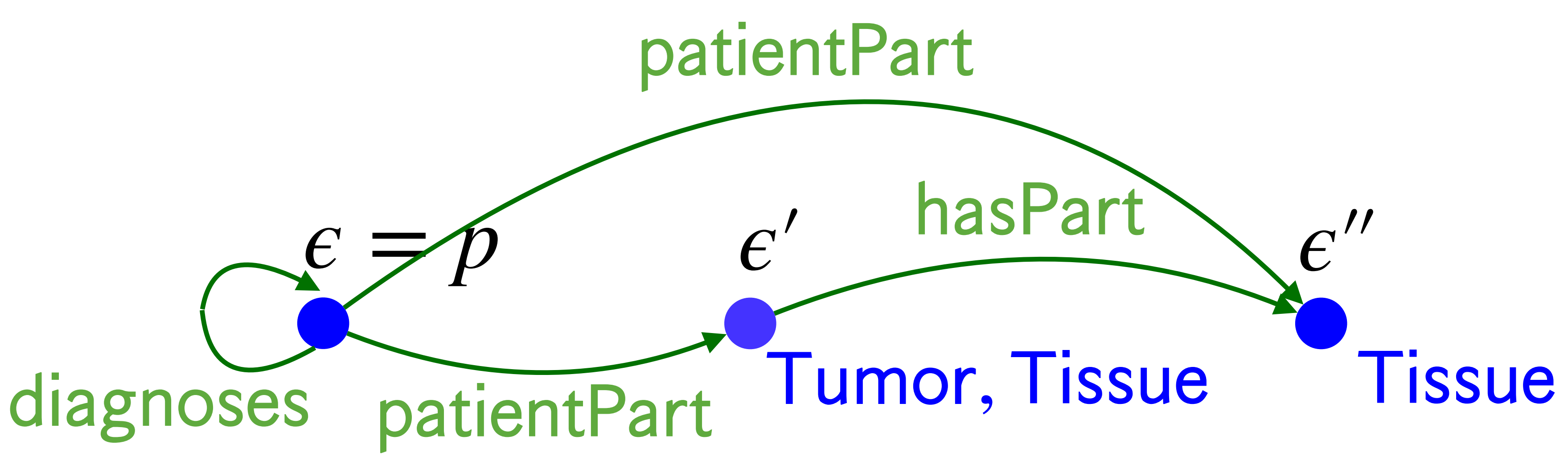
Tissue $\exists \mathsf{diagnoses}\,.\,\mathsf{Self}$
$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $(\mathsf{patientPart} \circ \mathsf{hasPart} \sqsubseteq \mathsf{patientPart})$

Semantics: $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I} \rangle$



Towards Standpoint-$\mathscr{E}\mathscr{L}$+

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I} \rangle$ of concept, role, individual

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self}$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue $\quad \exists \mathsf{diagnoses}\,.\,\mathsf{Self}$

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\quad \exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D\,,\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a),\quad r(a,b)$

$\big(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}\big)$ $\quad \big(\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}\big)(p)$

Towards Standpoint-$\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_{\mathsf{C}}, N_{\mathsf{R}}, N_{\mathrm{I}}, N_{\mathsf{S}} \rangle$ of concept, role, individual and standpoint names

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,Self$

With $A \in N_{\mathsf{C}}, r \in N_{\mathsf{R}}$

Tissue $\quad \exists \mathsf{diagnoses}\,.\,\mathsf{Self}$
$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\quad \exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D\,,\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a),\quad r(a, b)$

$\big(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}\big)$ $\quad \big(\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}\big)(p)$

Towards Standpoint-$\mathscr{E}\mathscr{L}$+

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S} \rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r \,.\, C \mid \exists r \,.\, \mathit{Self}$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}$

Tissue $\quad$ $\exists \mathsf{diagnoses}\,.\,\mathsf{Self}$

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\quad$ $\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a, b)$

$(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue})$ $\quad$ $(\exists \mathsf{patientPart}\,.\,\mathsf{Tumor})(p)$

Towards Standpoint-$\mathscr{E}\mathscr{L}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S} \rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self} \mid \odot_s C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$, $s \in N_\mathsf{S}$, $\odot \in \{\Box, \Diamond\}$.

Tissue $\quad \exists\mathsf{diagnoses}\,.\,\mathsf{Self}$

Process $\sqcap$ Tissue $\quad \exists\mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D\,, \quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a), \quad r(a, b)$

$\left(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}\right) \qquad \left(\exists\mathsf{patientPart}\,.\,\mathsf{Tumor}\right)(p)$

Towards Standpoint-$\mathcal{EL}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S} \rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self} \mid \odot_s C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$, $s \in N_\mathsf{S}$, $\odot \in \{\Box, \Diamond\}$.

Tissue $\quad \exists \mathsf{diagnoses}\,.\,\mathsf{Self}$ $\quad \Diamond_\mathsf{S}\,\mathsf{Process}$

$\mathsf{Process} \sqcap \mathsf{Tissue}$ $\quad \exists \mathsf{patientPart}\,.\,\mathsf{Tumor}$

The **set of axioms** includes:

- GCIs and RIAs: $C \sqsubseteq D$, $\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a, b)$

$\big(\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}\big)$ $\quad \big(\exists \mathsf{patientPart}\,.\,\mathsf{Tumor}\big)(p)$

Towards Standpoint-$\mathscr{EL}+$

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S} \rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self} \mid \odot_s C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$, $s \in N_\mathsf{S}$, $\odot \in \{\Box, \Diamond\}$.

Tissue $\quad \exists$diagnoses . Self $\quad \Diamond_\mathsf{S}$ Process

Process $\sqcap$ Tissue $\quad \exists$patientPart . Tumor

Formulas are $\odot_s (\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

- GCIs and RIAs: $C \sqsubseteq D$, $\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a, b)$

$\Box_\mathsf{L} \big((\text{Tumor} \sqsubseteq \text{Tissue}) \wedge \neg (\exists \text{patientPart}\,.\,\text{Tumor})(p) \big)$

Towards Standpoint-$\mathcal{EL}$+

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S}\rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r.C \mid \exists r.\mathit{Self} \mid \odot_s C$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$, $s \in N_\mathsf{S}$, $\odot \in \{\Box, \Diamond\}$.

Tissue $\quad \exists$diagnoses . Self $\quad \Diamond_\mathsf{S}$ Process

Process $\sqcap$ Tissue $\quad \exists$patientPart . Tumor

Formulas are $\odot_s(\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

$\Box_\mathsf{L}\big((\text{Tumor} \sqsubseteq \text{Tissue}) \wedge \neg(\exists\text{patientPart}.\text{Tumor})(p)\big)$

Semantics: $\mathscr{D} = \langle \Delta, \Pi, \sigma, \gamma \rangle$

Towards Standpoint-$\mathscr{EL}$+

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S} \rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,Self \mid \odot_s C$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}, s \in N_\mathsf{S}, \odot \in \{\Box, \Diamond\}$.

Tissue $\quad \exists\mathsf{diagnoses}\,.\,\mathsf{Self} \quad \Diamond_\mathsf{S}\,\mathsf{Process}$

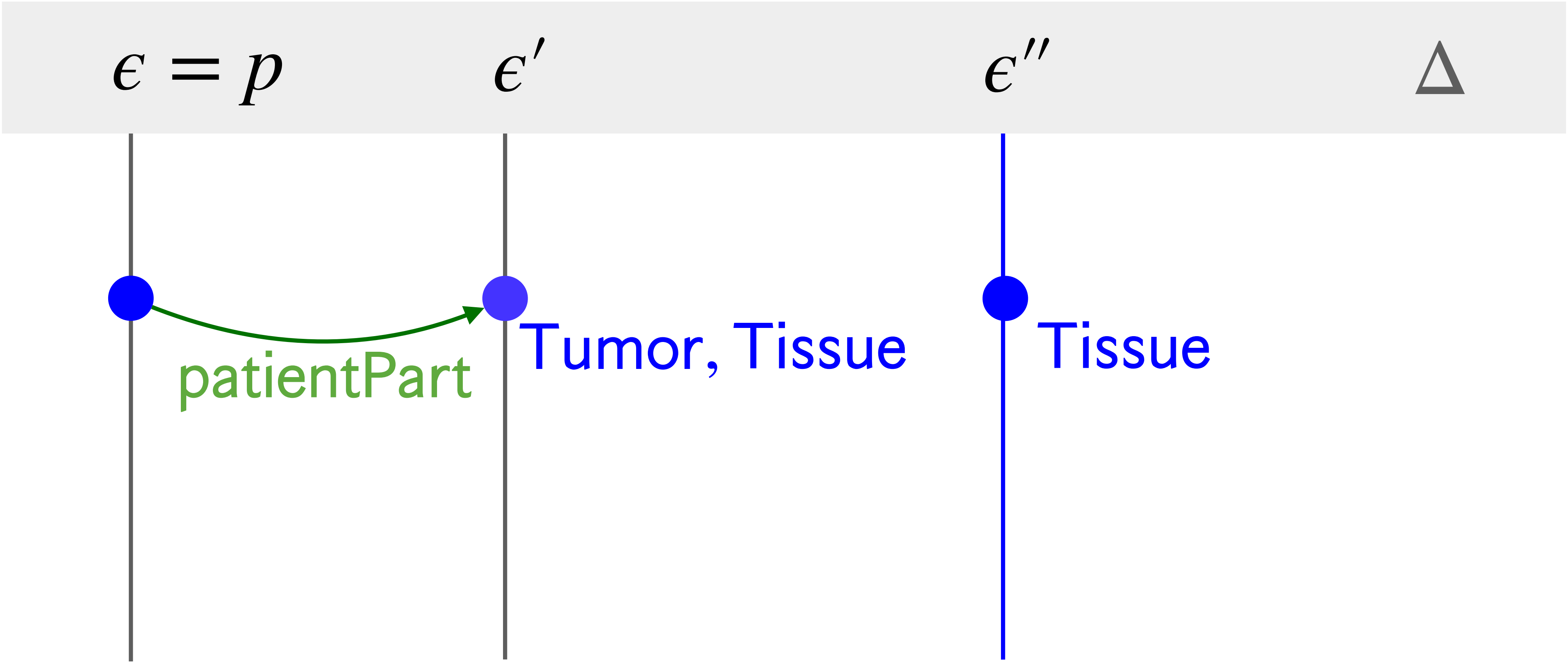
$\mathsf{Process} \sqcap \mathsf{Tissue} \quad \exists\mathsf{patientPart}\,.\,\mathsf{Tumor}$

Formulas are $\odot_s\,(\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

- GCIs and RIAs: $C \sqsubseteq D$, $\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a, b)$

$\Box_\mathsf{L}\Big((\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}) \wedge \neg\big(\exists\mathsf{patientPart}\,.\,\mathsf{Tumor}\big)(p)\Big)$

Semantics: $\mathscr{D} = \langle \Delta, \Pi, \sigma, \gamma \rangle$



Towards Standpoint-$\mathscr{EL}$+

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S} \rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self} \mid \odot_s C$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}, s \in N_\mathsf{S}, \odot \in \{\Box, \Diamond\}$.

Tissue $\exists$diagnoses . Self $\Diamond_\mathsf{S}$ Process

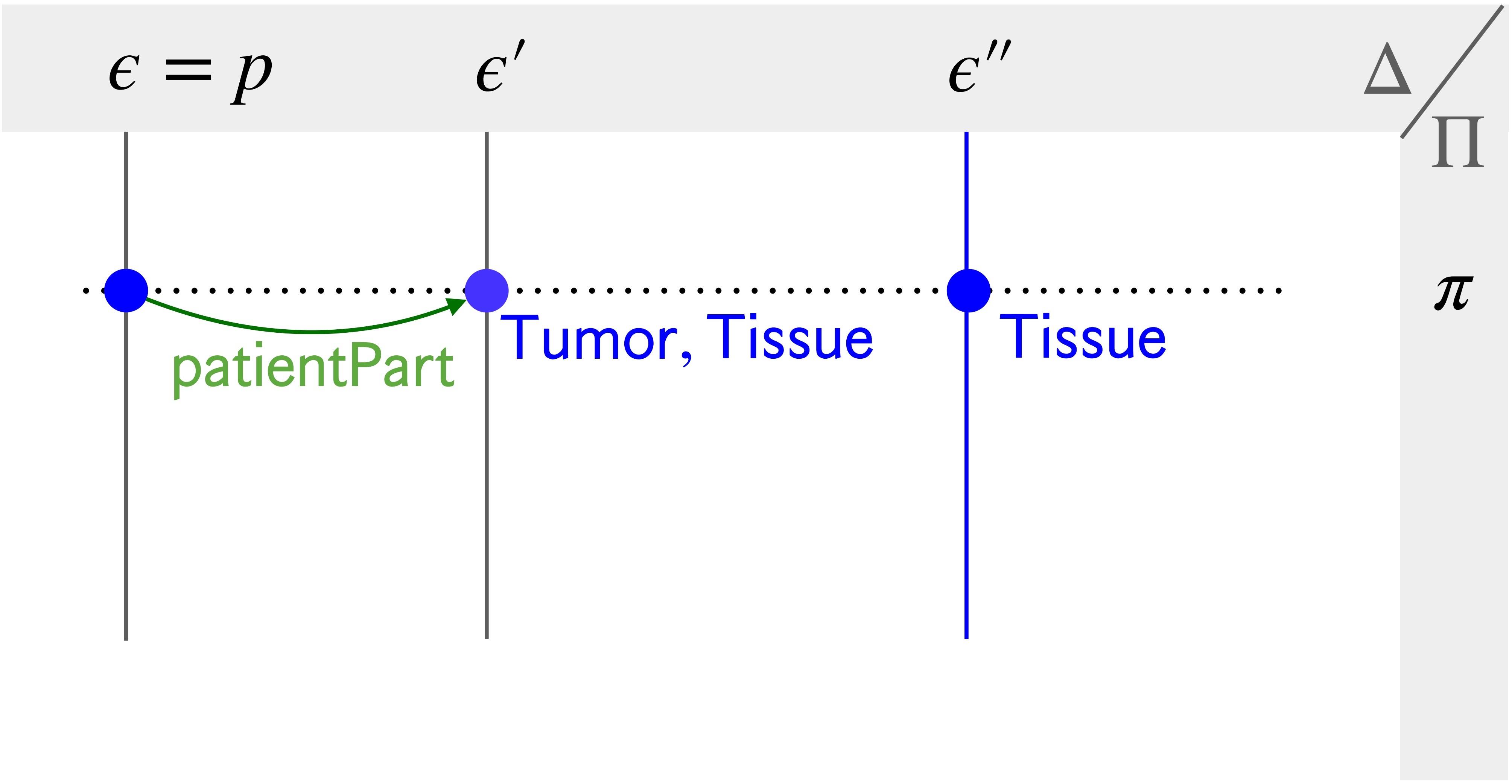
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

Formulas are $\odot_s (\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

$\Box_\mathsf{L}\Big((\mathsf{Tumor} \sqsubseteq \mathsf{Tissue}) \wedge \neg\,(\exists \mathsf{patientPart}\,.\,\mathsf{Tumor})(p)\Big)$

Semantics: $\mathscr{D} = \langle \Delta, \Pi, \sigma, \gamma \rangle$



Towards Standpoint-$\mathcal{EL}$+

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S} \rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,Self \mid \odot_s C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$, $s \in N_\mathsf{S}$, $\odot \in \{\Box, \Diamond\}$.

Tissue $\quad \exists$diagnoses . Self $\quad \Diamond_\mathsf{S}$ Process

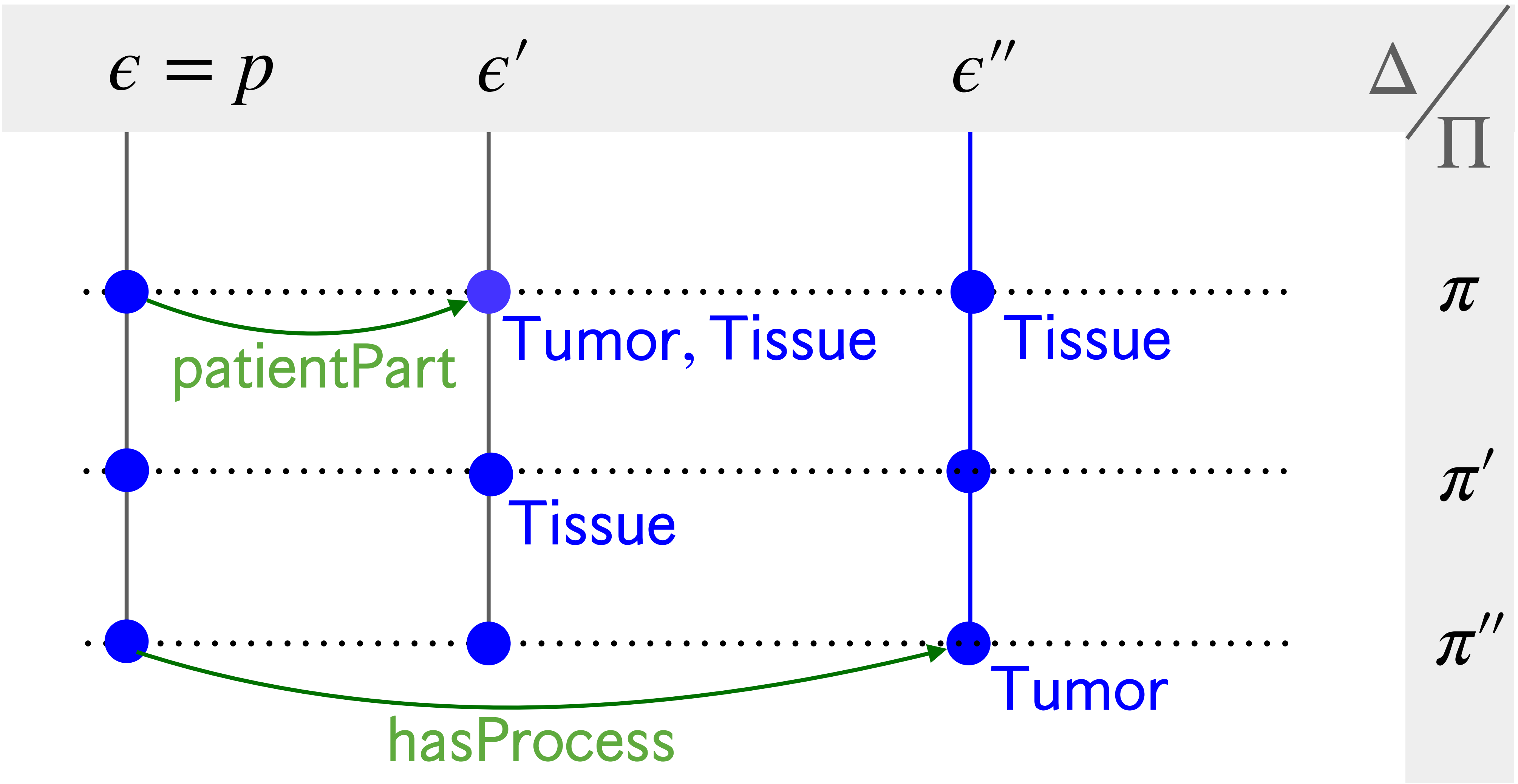
Process $\sqcap$ Tissue $\quad \exists$patientPart . Tumor

Formulas are $\odot_s (\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

- GCIs and RIAs: $C \sqsubseteq D$, $\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a,b)$

$\Box_\mathsf{L}\Big((\text{Tumor} \sqsubseteq \text{Tissue}) \wedge \neg\big(\exists \text{patientPart}\,.\,\text{Tumor}\big)(p)\Big)$

Semantics: $\mathscr{D} = \langle \Delta, \Pi, \sigma, \gamma \rangle$



Towards Standpoint-$\mathscr{EL}$+

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S}\rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self} \mid \odot_s C$$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$, $s \in N_\mathsf{S}$, $\odot \in \{\Box, \Diamond\}$.

Tissue ∃diagnoses . Self $\Diamond_\mathsf{S}$ Process

Process ⊓ Tissue ∃patientPart . Tumor

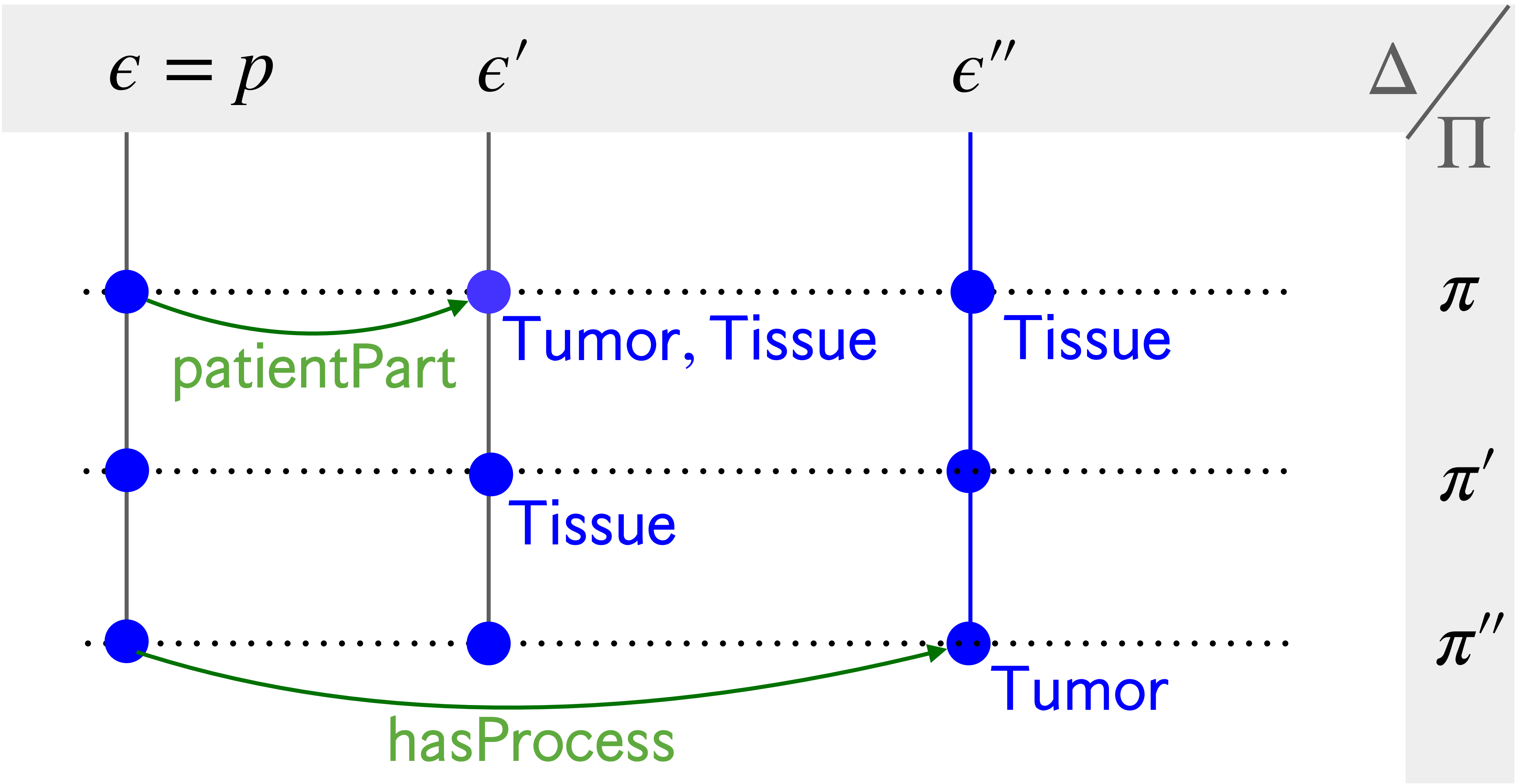
Formulas are $\odot_s(\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a, b)$

$\Box_\mathsf{L}\Big((\text{Tumor} \sqsubseteq \text{Tissue}) \wedge \neg(\exists \text{patientPart}\,.\,\text{Tumor})(p)\Big)$

Semantics: $\mathscr{D} = \langle \Delta, \Pi, \sigma, \gamma\rangle$

- γ maps each $\pi \in \Pi$ to an $\mathscr{EL}$+ interpretation $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I}\rangle$



Towards Standpoint-$\mathscr{EL}$+

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S}\rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self} \mid \odot_s C$$

With $A \in N_\mathsf{C}, r \in N_\mathsf{R}, s \in N_\mathsf{S}, \odot \in \{\Box, \Diamond\}$.

Tissue $\quad \exists$diagnoses . Self $\quad \Diamond_\mathsf{S}$ Process

Process $\sqcap$ Tissue $\quad \exists$patientPart . Tumor

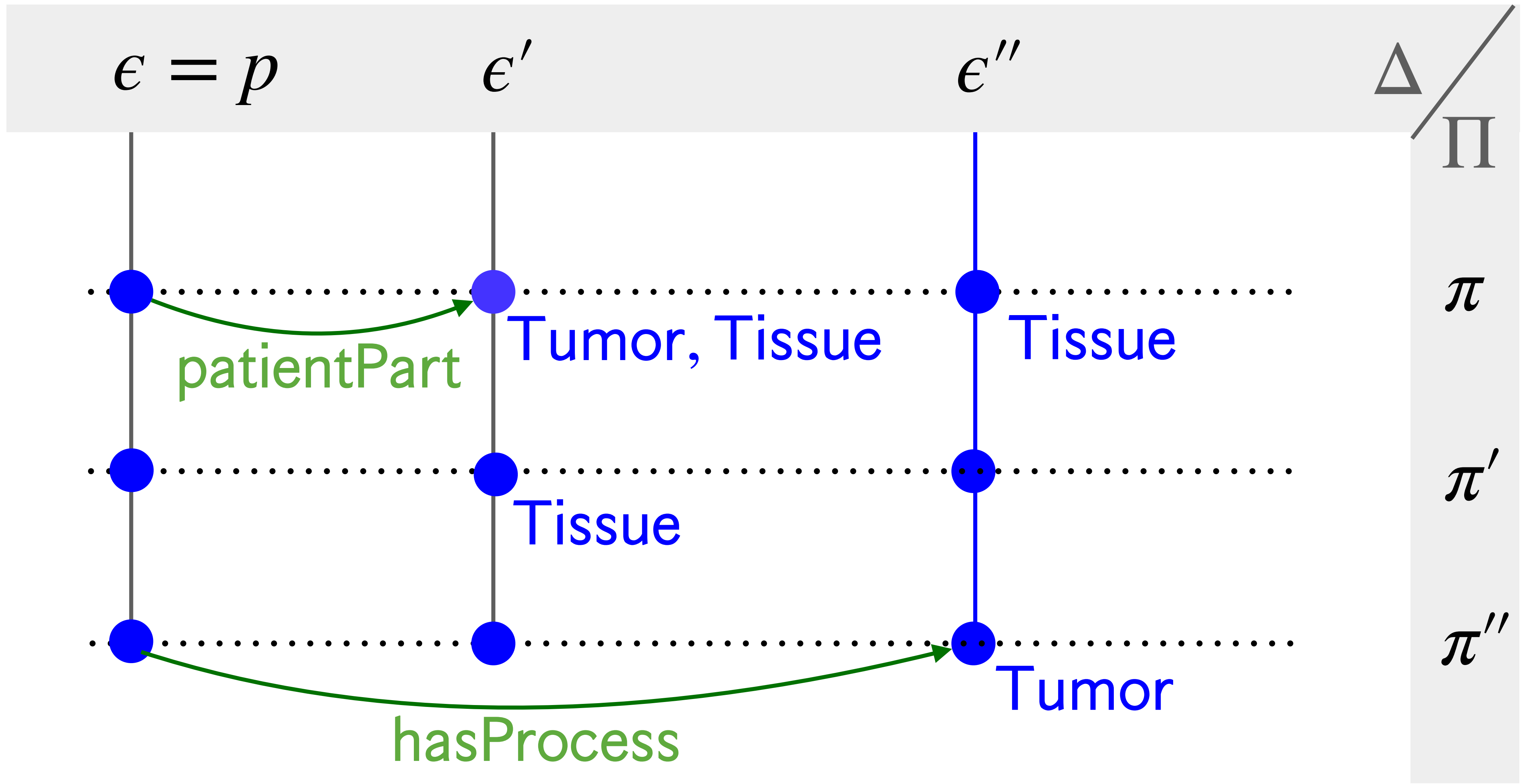
Formulas are $\odot_s(\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

- GCIs and RIAs: $C \sqsubseteq D$, $\quad R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $\quad r(a, b)$

$\Box_\mathsf{L}\big((\text{Tumor} \sqsubseteq \text{Tissue}) \wedge \neg(\exists \text{patientPart}\,.\,\text{Tumor})(p)\big)$

Semantics: $\mathscr{D} = \langle \Delta, \Pi, \sigma, \gamma\rangle$

- γ maps each $\pi \in \Pi$ to an $\mathscr{EL}$+ interpretation $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I}\rangle$
- σ maps each $s \in N_\mathsf{S}$ to a **nonempty** subset of Π



Towards Standpoint-$\mathscr{E}\mathscr{L}$+

Vocabulary $\langle N_\mathsf{C}, N_\mathsf{R}, N_\mathsf{I}, N_\mathsf{S}\rangle$ of concept, role, individual and standpoint names, $* \in N_\mathsf{S}$ (universal standpoint).

Syntax:

The **set of concepts** is given by

$C ::= \top \mid \bot \mid A \mid C_1 \sqcap C_2 \mid \exists r\,.\,C \mid \exists r\,.\,\mathit{Self} \mid \odot_s C$

With $A \in N_\mathsf{C}$, $r \in N_\mathsf{R}$, $s \in N_\mathsf{S}$, $\odot \in \{\Box, \Diamond\}$.

Tissue $\exists$diagnoses . Self $\Diamond_\mathsf{S}$ Process
Process $\sqcap$ Tissue $\exists$patientPart . Tumor

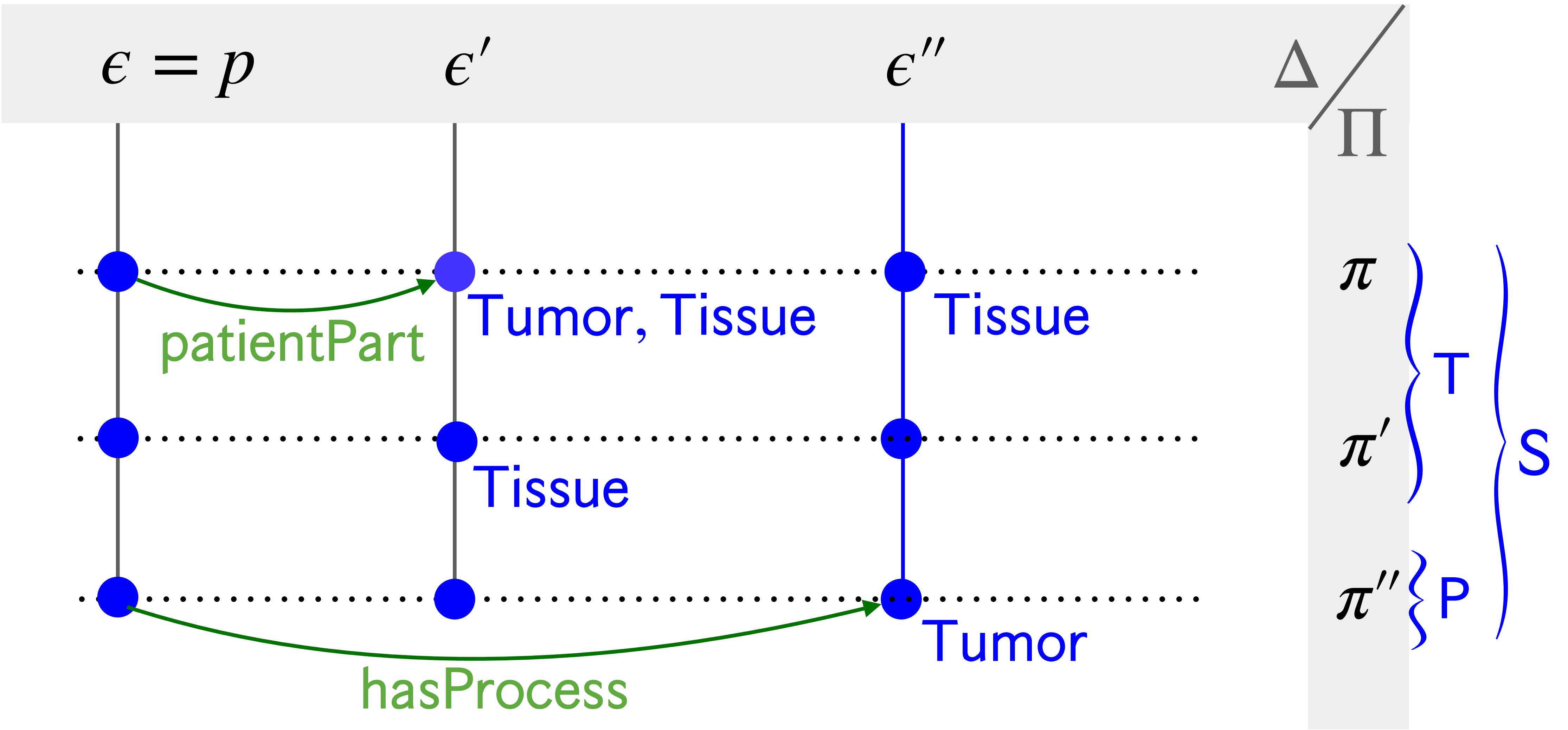
Formulas are $\odot_s(\lambda_1 \wedge \ldots \wedge \lambda_n)$ for $\lambda_i \in \{\mathscr{E}, \neg\mathscr{E}\}$, $\mathscr{E}$:

- GCIs and RIAs: $C \sqsubseteq D$, $R_1 \circ \ldots \circ R_n \sqsubseteq R$
- Assertions: $C(a)$, $r(a,b)$

$\Box_\mathsf{L}\Big((\text{Tumor} \sqsubseteq \text{Tissue}) \wedge \neg\big(\exists\text{patientPart}\,.\,\text{Tumor}\big)(p)\Big)$

Semantics: $\mathscr{D} = \langle \Delta, \Pi, \sigma, \gamma \rangle$

- γ maps each $\pi \in \Pi$ to an $\mathscr{E}\mathscr{L}$+ interpretation $\mathcal{I} = \langle \Delta, \cdot^\mathcal{I}\rangle$
- σ maps each $s \in N_\mathsf{S}$ to a **nonempty** subset of Π



Complexity and Automated Reasoning

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}^+}$

Decision Calculus for $\mathbb{S}_{\mathcal{EL}+}$

(1) Normalisation:

- Sharpenings:
  - $\mathsf{s}' \preceq \mathsf{s}$ $\quad$ $\mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}$
- GCIs:
  - $\Box_{\mathsf{s}}(C \sqsubseteq D)$ $\quad$ $\Box_{\mathsf{s}}(C_1 \sqcap C_2 \sqsubseteq D)$
  - $\Box_{\mathsf{s}}(\exists r . C \sqsubseteq D)$ $\quad$ $\Box_{\mathsf{s}}(C \sqsubseteq \exists r . D)$
  - $\Box_{\mathsf{s}}(C \sqsubseteq \Box_{\mathsf{u}} D)$ $\quad$ $\Box_{\mathsf{s}}(C \sqsubseteq \Diamond_{\mathsf{u}} D)$
- RIAs:
  - $\Box_{\mathsf{s}}(R' \sqsubseteq R)$ $\quad$ $\Box_{\mathsf{s}}(R_1 \circ R_2 \sqsubseteq R)$
- Concept and role assertions:
  - $\Box_{\mathsf{s}} C(a)$ $\quad$ $\Box_{\mathsf{s}} r(a, b)$

Decision Calculus for $\mathbb{S}_{\mathcal{EL}+}$

(1) Normalisation:

- Sharpenings:
  - $\mathsf{s}' \preceq \mathsf{s}$ $\qquad$ $\mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}$
- GCIs:
  - $\Box_{\mathsf{s}}(C \sqsubseteq D)$ $\qquad$ $\Box_{\mathsf{s}}(C_1 \sqcap C_2 \sqsubseteq D)$
  - $\Box_{\mathsf{s}}(\exists r . C \sqsubseteq D)$ $\qquad$ $\Box_{\mathsf{s}}(C \sqsubseteq \exists r . D)$
  - $\Box_{\mathsf{s}}(C \sqsubseteq \Box_{\mathsf{u}} D)$ $\qquad$ $\Box_{\mathsf{s}}(C \sqsubseteq \Diamond_{\mathsf{u}} D)$
- RIAs:
  - $\Box_{\mathsf{s}}(R' \sqsubseteq R)$ $\qquad$ $\Box_{\mathsf{s}}(R_1 \circ R_2 \sqsubseteq R)$
- Concept and role assertions:
  - $\Box_{\mathsf{s}}\, C(a)$ $\qquad$ $\Box_{\mathsf{s}}\, r(a, b)$

(2) Extended modalised GCIs:

$$\Box_{\mathsf{t}}\,[A \sqsubseteq \Box_{\mathsf{s}}\,[B \Rightarrow C]]$$

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

(1) Normalisation:

- Sharpenings:
  - $\mathsf{s}' \preceq \mathsf{s}$ $\quad$ $\mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}$
- GCIs:
  - $\square_\mathsf{s}(C \sqsubseteq D)$ $\quad$ $\square_\mathsf{s}(C_1 \sqcap C_2 \sqsubseteq D)$
  - $\square_\mathsf{s}(\exists r.C \sqsubseteq D)$ $\quad$ $\square_\mathsf{s}(C \sqsubseteq \exists r.D)$
  - $\square_\mathsf{s}(C \sqsubseteq \square_\mathsf{u} D)$ $\quad$ $\square_\mathsf{s}(C \sqsubseteq \lozenge_\mathsf{u} D)$
- RIAs:
  - $\square_\mathsf{s}(R' \sqsubseteq R)$ $\quad$ $\square_\mathsf{s}(R_1 \circ R_2 \sqsubseteq R)$
- Concept and role assertions:
  - $\square_\mathsf{s} C(a)$ $\quad$ $\square_\mathsf{s} r(a,b)$

(2) Extended modalised GCIs:

$$\square_\mathsf{t}[A \sqsubseteq \square_\mathsf{s}[B \Rightarrow C]]$$

* can be rewritten (with a fresh concept D) to
$\square_\mathsf{t}[A \sqsubseteq \square_\mathsf{s} D]$ and $\square_\mathsf{s}[D \sqcap B \sqsubseteq C]]$

Decision Calculus for $\mathbb{S}_{\mathcal{EL}+}$

(1) Normalisation:

- Sharpenings:
  - $s' \preceq s$ $\quad$ $s_1 \cap s_2 \preceq s$
- GCIs:
  - $\square_s (C \sqsubseteq D)$ $\quad$ $\square_s (C_1 \sqcap C_2 \sqsubseteq D)$
  - $\square_s (\exists r . C \sqsubseteq D)$ $\quad$ $\square_s (C \sqsubseteq \exists r . D)$
  - $\square_s (C \sqsubseteq \square_u D)$ $\quad$ $\square_s (C \sqsubseteq \Diamond_u D)$
- RIAs:
  - $\square_s (R' \sqsubseteq R)$ $\quad$ $\square_s (R_1 \circ R_2 \sqsubseteq R)$
- Concept and role assertions:
  - $\square_s C(a)$ $\quad$ $\square_s r(a, b)$

(2) Extended modalised GCIs:

$$\square_t [A \sqsubseteq \square_s [B \Rightarrow C]]$$

* can be rewritten (with a fresh concept D) to
  $\square_t [A \sqsubseteq \square_s D]$ and $\square_s [D \sqcap B \sqsubseteq C]]$

Then replace:

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

(1) Normalisation:

- Sharpenings:
  - $\mathsf{s}' \preceq \mathsf{s}$ $\qquad$ $\mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}$
- GCIs:
  - $\square_{\mathsf{s}}(C \sqsubseteq D)$ $\qquad$ $\square_{\mathsf{s}}(C_1 \sqcap C_2 \sqsubseteq D)$
  - $\square_{\mathsf{s}}(\exists r\,.\,C \sqsubseteq D)$ $\qquad$ $\square_{\mathsf{s}}(C \sqsubseteq \exists r\,.\,D)$
  - $\square_{\mathsf{s}}(C \sqsubseteq \square_{\mathsf{u}} D)$ $\qquad$ $\square_{\mathsf{s}}(C \sqsubseteq \lozenge_{\mathsf{u}} D)$
- RIAs:
  - $\square_{\mathsf{s}}(R' \sqsubseteq R)$ $\qquad$ $\square_{\mathsf{s}}(R_1 \circ R_2 \sqsubseteq R)$
- Concept and role assertions:
  - $\square_{\mathsf{s}}\, C(a)$ $\qquad$ $\square_{\mathsf{s}}\, r(a,b)$

(2) Extended modalised GCIs:

$$\square_{\mathsf{t}}\,[A \sqsubseteq \square_{\mathsf{s}}\,[B \Rightarrow C]]$$

* can be rewritten (with a fresh concept D) to

$\square_{\mathsf{t}}\,[A \sqsubseteq \square_{\mathsf{s}}\, D]$ and $\square_{\mathsf{s}}\,[D \sqcap B \sqsubseteq C]]$

Then replace:

- $\square_{\mathsf{s}}(C \sqsubseteq D)$ by $\square_{*}\,[\,\top \sqsubseteq \square_{\mathsf{s}}\,[C \Rightarrow D]]$

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

(1) Normalisation:

- Sharpenings:
  - $\mathsf{s}' \preceq \mathsf{s}$ $\quad$ $\mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}$
- GCIs:
  - $\Box_{\mathsf{s}}(C \sqsubseteq D)$ $\quad$ $\Box_{\mathsf{s}}(C_1 \sqcap C_2 \sqsubseteq D)$
  - $\Box_{\mathsf{s}}(\exists r\,.\,C \sqsubseteq D)$ $\quad$ $\Box_{\mathsf{s}}(C \sqsubseteq \exists r\,.\,D)$
  - $\Box_{\mathsf{s}}(C \sqsubseteq \Box_{\mathsf{u}} D)$ $\quad$ $\Box_{\mathsf{s}}(C \sqsubseteq \Diamond_{\mathsf{u}} D)$
- RIAs:
  - $\Box_{\mathsf{s}}(R' \sqsubseteq R)$ $\quad$ $\Box_{\mathsf{s}}(R_1 \circ R_2 \sqsubseteq R)$
- Concept and role assertions:
  - $\Box_{\mathsf{s}}\, C(a)$ $\quad$ $\Box_{\mathsf{s}}\, r(a, b)$

(2) Extended modalised GCIs:

$$\Box_{\mathsf{t}}\,[A \sqsubseteq \Box_{\mathsf{s}}\,[B \Rightarrow C]]$$

* can be rewritten (with a fresh concept D) to

$\Box_{\mathsf{t}}\,[A \sqsubseteq \Box_{\mathsf{s}}\, D]$ and $\Box_{\mathsf{s}}\,[D \sqcap B \sqsubseteq C]]$

Then replace:

- $\Box_{\mathsf{s}}(C \sqsubseteq D)$ by $\Box_{*}\,[\top \sqsubseteq \Box_{\mathsf{s}}\,[C \Rightarrow D]]$
- $\Box_{\mathsf{s}}(C \sqsubseteq \Box_{\mathsf{u}} D)$ by $\Box_{\mathsf{s}}\,[C \sqsubseteq \Box_{\mathsf{u}}\,[\top \Rightarrow D]]$

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

(1) Normalisation:

- Sharpenings:
  - $s' \preceq s$ $\quad s_1 \cap s_2 \preceq s$
- GCIs:
  - $\square_s (C \sqsubseteq D)$ $\quad \square_s (C_1 \sqcap C_2 \sqsubseteq D)$
  - $\square_s (\exists r . C \sqsubseteq D)$ $\quad \square_s (C \sqsubseteq \exists r . D)$
  - $\square_s (C \sqsubseteq \square_u D)$ $\quad \square_s (C \sqsubseteq \Diamond_u D)$
- RIAs:
  - $\square_s (R' \sqsubseteq R)$ $\quad \square_s (R_1 \circ R_2 \sqsubseteq R)$
- Concept and role assertions:
  - $\square_s C(a)$ $\quad \square_s r(a, b)$

(2) Extended modalised GCIs:

$$\square_t [A \sqsubseteq \square_s [B \Rightarrow C]]$$

* can be rewritten (with a fresh concept D) to
$\square_t [A \sqsubseteq \square_s D]$ and $\square_s [D \sqcap B \sqsubseteq C]]$

Then replace:

- $\square_s (C \sqsubseteq D)$ by $\square_* [\top \sqsubseteq \square_s [C \Rightarrow D]]$
- $\square_s (C \sqsubseteq \square_u D)$ by $\square_s [C \sqsubseteq \square_u [\top \Rightarrow D]]$
- $\square_s C(a)$ by $\square_s [\{a\} \sqsubseteq \square_s [\top \Rightarrow C]]$

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

(1) Normalisation:

- Sharpenings:
  - $\mathsf{s}' \preceq \mathsf{s}$ $\quad$ $\mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}$
- GCIs:
  - $\square_{\mathsf{s}}(C \sqsubseteq D)$ $\quad$ $\square_{\mathsf{s}}(C_1 \sqcap C_2 \sqsubseteq D)$
  - $\square_{\mathsf{s}}(\exists r.C \sqsubseteq D)$ $\quad$ $\square_{\mathsf{s}}(C \sqsubseteq \exists r.D)$
  - $\square_{\mathsf{s}}(C \sqsubseteq \square_{\mathsf{u}} D)$ $\quad$ $\square_{\mathsf{s}}(C \sqsubseteq \lozenge_{\mathsf{u}} D)$
- RIAs:
  - $\square_{\mathsf{s}}(R' \sqsubseteq R)$ $\quad$ $\square_{\mathsf{s}}(R_1 \circ R_2 \sqsubseteq R)$
- Concept and role assertions:
  - $\square_{\mathsf{s}} C(a)$ $\quad$ $\square_{\mathsf{s}} r(a,b)$

(2) Extended modalised GCIs:

$$\square_{\mathsf{t}}[A \sqsubseteq \square_{\mathsf{s}}[B \Rightarrow C]]$$

* can be rewritten (with a fresh concept D) to
$\square_{\mathsf{t}}[A \sqsubseteq \square_{\mathsf{s}} D]$ and $\square_{\mathsf{s}}[D \sqcap B \sqsubseteq C]]$

Then replace:

- $\square_{\mathsf{s}}(C \sqsubseteq D)$ by $\square_{*}[\top \sqsubseteq \square_{\mathsf{s}}[C \Rightarrow D]]$
- $\square_{\mathsf{s}}(C \sqsubseteq \square_{\mathsf{u}} D)$ by $\square_{\mathsf{s}}[C \sqsubseteq \square_{\mathsf{u}}[\top \Rightarrow D]]$
- $\square_{\mathsf{s}} C(a)$ by $\square_{\mathsf{s}}[\{a\} \sqsubseteq \square_{\mathsf{s}}[\top \Rightarrow C]]$

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

Tautologies

$$(T.1)\ \frac{}{\mathsf{s} \preceq *} \qquad (T.2)\ \frac{}{\mathsf{s} \preceq \mathsf{s}} \qquad (T.3)\ \frac{}{\square_*[\top \sqsubseteq \square_*[C \Rightarrow C]]} \qquad (T.4)\ \frac{}{\square_*[\top \sqsubseteq \square_*[C \Rightarrow \top]]} \qquad (T.5)\ \frac{}{\square_*[R \sqsubseteq R]}$$

Standpoint hierarchy rules (for all $\mathsf{s} \in N_\mathsf{S}$, ξ being any extended GCI, RIA, or role assertion)

$$(S.1)\ \frac{\mathsf{s} \preceq \mathsf{s}' \quad \mathsf{s}' \preceq \mathsf{s}''}{\mathsf{s} \preceq \mathsf{s}''} \qquad (S.2)\ \frac{\mathsf{s} \preceq \mathsf{s}_1 \quad \mathsf{s} \preceq \mathsf{s}_2 \quad \mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}'}{\mathsf{s} \preceq \mathsf{s}'} \qquad (S.3)\ \frac{\square_{\mathsf{s}'}\xi \quad \mathsf{s} \preceq \mathsf{s}'}{\square_\mathsf{s}\xi} \qquad (S.4)\ \frac{\square_\mathsf{t}[C \sqsubseteq \square_{\mathsf{s}'}[D \Rightarrow E]] \quad \mathsf{s} \preceq \mathsf{s}'}{\square_\mathsf{t}[C \sqsubseteq \square_\mathsf{s}[D \Rightarrow E]]}$$

Internal inferences for extended GCIs

$$(I.1)\ \frac{\square_\mathsf{s}[C \sqsubseteq \square_\mathsf{s}[\top \Rightarrow D]]}{\square_*[\top \sqsubseteq \square_\mathsf{s}[C \Rightarrow D]]} \qquad (I.2)\ \frac{\square_\mathsf{u}[\top \sqsubseteq \square_\mathsf{s}[C \Rightarrow D]]}{\square_*[\top \sqsubseteq \square_\mathsf{s}[C \Rightarrow D]]}$$

Role subsumptions

$$(R.1)\ \frac{\square_\mathsf{s}[R \sqsubseteq R''] \quad \square_\mathsf{s}[R'' \sqsubseteq R']}{\square_\mathsf{s}[R \sqsubseteq R']}$$

Forward chaining

$$(C.1)\ \frac{\square_\mathsf{t}[B \sqsubseteq \square_\mathsf{s}[C \Rightarrow D]] \quad \square_\mathsf{t}[B \sqsubseteq \square_\mathsf{s}[D \Rightarrow E]]}{\square_\mathsf{t}[B \sqsubseteq \square_\mathsf{s}[C \Rightarrow E]]} \qquad (C.2)\ \frac{\square_\mathsf{u}[\top \sqsubseteq \square_\mathsf{t}[B \Rightarrow C]] \quad \square_\mathsf{t}[C \sqsubseteq \square_\mathsf{s}[D \Rightarrow E]]}{\square_\mathsf{t}[B \sqsubseteq \square_\mathsf{s}[D \Rightarrow E]]}$$

$$(C.3)\ \frac{\square_\mathsf{u}[\top \sqsubseteq \square_\mathsf{t}[C \Rightarrow D]] \quad \square_\mathsf{t}[D \sqsubseteq \lozenge_\mathsf{s}E]}{\square_\mathsf{t}[C \sqsubseteq \lozenge_\mathsf{s}E]} \qquad (C.4)\ \frac{\square_\mathsf{t}[C \sqsubseteq \lozenge_\mathsf{s}D] \quad \square_\mathsf{t}[C \sqsubseteq \square_\mathsf{s}[D \Rightarrow E]]}{\square_\mathsf{t}[C \sqsubseteq \lozenge_\mathsf{s}E]}$$

... (26 more rules)

Decision Calculus for $\mathbb{S}_{\mathcal{EL}+}$

Tautologies

$$(T.1)\ \frac{}{\mathsf{s} \preceq *} \qquad (T.2)\ \frac{}{\mathsf{s} \preceq \mathsf{s}} \qquad (T.3)\ \frac{}{\square_*[\top \sqsubseteq \square_*[C \Rightarrow C]]} \qquad (T.4)\ \frac{}{\square_*[\top \sqsubseteq \square_*[C \Rightarrow \top]]} \qquad (T.5)\ \frac{}{\square_*[R \sqsubseteq R]}$$

Standpoint hierarchy rules (for all $\mathsf{s} \in N_\mathsf{S}$, ξ being any extended GCI, RIA, or role assertion)

$$(S.1)\ \frac{\mathsf{s} \preceq \mathsf{s}' \quad \mathsf{s}' \preceq \mathsf{s}''}{\mathsf{s} \preceq \mathsf{s}''} \qquad (S.2)\ \frac{\mathsf{s} \preceq \mathsf{s}_1 \quad \mathsf{s} \preceq \mathsf{s}_2 \quad \mathsf{s}_1 \cap \mathsf{s}_2 \preceq \mathsf{s}'}{\mathsf{s} \preceq \mathsf{s}'} \qquad (S.3)\ \frac{\square_{\mathsf{s}'}\xi \quad \mathsf{s} \preceq \mathsf{s}'}{\square_\mathsf{s}\xi} \qquad (S.4)\ \frac{\square_\mathsf{t}[C \sqsubseteq \square_{\mathsf{s}'}[D \Rightarrow E]] \quad \mathsf{s} \preceq \mathsf{s}'}{\square_\mathsf{t}[C \sqsubseteq \square_\mathsf{s}[D \Rightarrow E]]}$$

Internal inferences for extended GCIs

$$(I.1)\ \frac{\square_\mathsf{s}[C \sqsubseteq \square_\mathsf{s}[\top \Rightarrow D]]}{\square_*[\top \sqsubseteq \square_\mathsf{s}[C \Rightarrow D]]} \qquad (I.2)\ \frac{\square_\mathsf{u}[\top \sqsubseteq \square_\mathsf{s}[C \Rightarrow D]]}{\square_*[\top \sqsubseteq \square_\mathsf{s}[C \Rightarrow D]]}$$

Role subsumptions

$$(R.1)\ \frac{\square_\mathsf{s}[R \sqsubseteq R''] \quad \square_\mathsf{s}[R'' \sqsubseteq R']}{\square_\mathsf{s}[R \sqsubseteq R']}$$

Forward chaining

If $\square_* [\ \top \sqsubseteq \square_* [\ \top \Rightarrow \bot\ ]] \notin \mathscr{K}^{\vdash}$, then $\mathscr{K}$ is satisfiable

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ (Proofs)

Decision Calculus for $\mathbb{S}_{\mathcal{EL}+}$ (Proofs)

Theorem 4 (Termination). *The closure of $\mathbb{S}_{\mathcal{EL}+}$ knowledge bases under the deduction calculus can be computed in PTIME.*

Decision Calculus for $\mathbb{S}_{\mathcal{EL}+}$ (Proofs)

Theorem 4 (Termination). *The closure of $\mathbb{S}_{\mathcal{EL}+}$ knowledge bases under the deduction calculus can be computed in PTIME.*

- Polynomial normalisation & worst-case optimal Datalog encoding of the saturation procedure.

Decision Calculus for $\mathbb{S}_{\mathcal{EL}+}$ (Proofs)

Theorem 4 (Termination). *The closure of $\mathbb{S}_{\mathcal{EL}+}$ knowledge bases under the deduction calculus can be computed in PTIME.*

- Polynomial normalisation & worst-case optimal Datalog encoding of the saturation procedure.

Theorem 5 (Soundness). *The deduction calculus is sound for $\mathbb{S}_{\mathcal{EL}+}$ knowledge bases.*

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ (Proofs)

Theorem 4 (Termination). *The closure of $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ knowledge bases under the deduction calculus can be computed in PTIME.*

- Polynomial normalisation & worst-case optimal Datalog encoding of the saturation procedure.

Theorem 5 (Soundness). *The deduction calculus is sound for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ knowledge bases.*

Theorem 6 (Completeness). *The deduction calculus is refutation-complete for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ knowledge bases.*

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ (Proofs)

Theorem 4 (Termination). *The closure of $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ knowledge bases under the deduction calculus can be computed in PTIME.*

- Polynomial normalisation & worst-case optimal Datalog encoding of the saturation procedure.

Theorem 5 (Soundness). *The deduction calculus is sound for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ knowledge bases.*

Theorem 6 (Completeness). *The deduction calculus is refutation-complete for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ knowledge bases.*

- We prove the existence of a model whenever $\square_* [\top \sqsubseteq \square_* [\top \Rightarrow \bot]] \notin \mathscr{K}^{\vdash}$.

Decision Calculus for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ (Proofs)

Theorem 4 (Termination). *The closure of $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ knowledge bases under the deduction calculus can be computed in PTIME.*

- Polynomial normalisation & worst-case optimal Datalog encoding of the saturation procedure.

Theorem 5 (Soundness). *The deduction calculus is sound for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ knowledge bases.*

Theorem 6 (Completeness). *The deduction calculus is refutation-complete for $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$ knowledge bases.*

- We prove the existence of a model whenever $\square_* [\, \top \sqsubseteq \square_* [\, \top \Rightarrow \bot \,]] \notin \mathscr{K}^{\vdash}$.
- This model is canonical in a sense but it will typically be infinite.

Tractable Reasoning in $\mathbb{S}_{\mathscr{EL}+}$

Tractable Reasoning in $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

- ➡ Complexity of the satisfiability of Standpoint-$\mathscr{E}\mathscr{L}$+ → PTime
- ➡ Tractability is easily lost:

Tractable Reasoning in $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

- ➡ Complexity of the satisfiability of Standpoint-$\mathscr{E}\mathscr{L}+$ $\rightarrow$ PTime
- ➡ Tractability is easily lost:
  - -Empty standpoints $\rightarrow$ NP-hard

Tractable Reasoning in $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

- ➡ Complexity of the satisfiability of Standpoint-$\mathscr{E}\mathscr{L}+$ $\rightarrow$ PTime
- ➡ Tractability is easily lost:
  - -Empty standpoints $\rightarrow$ NP-hard
  - -Rigid roles $\rightarrow$ CoNP-hard

Tractable Reasoning in $\mathbb{S}_{\mathscr{E}\mathscr{L}+}$

- ➡ Complexity of the satisfiability of Standpoint-$\mathscr{E}\mathscr{L}+$ $\rightarrow$ PTime
- ➡ Tractability is easily lost:
  - -Empty standpoints $\rightarrow$ NP-hard
  - -Rigid roles $\rightarrow$ CoNP-hard
  - -Nominal Concepts $\rightarrow$ ExpTime-hard

Future Research Goals and Challenges

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:

(O1) **Decidability and complexity**

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:

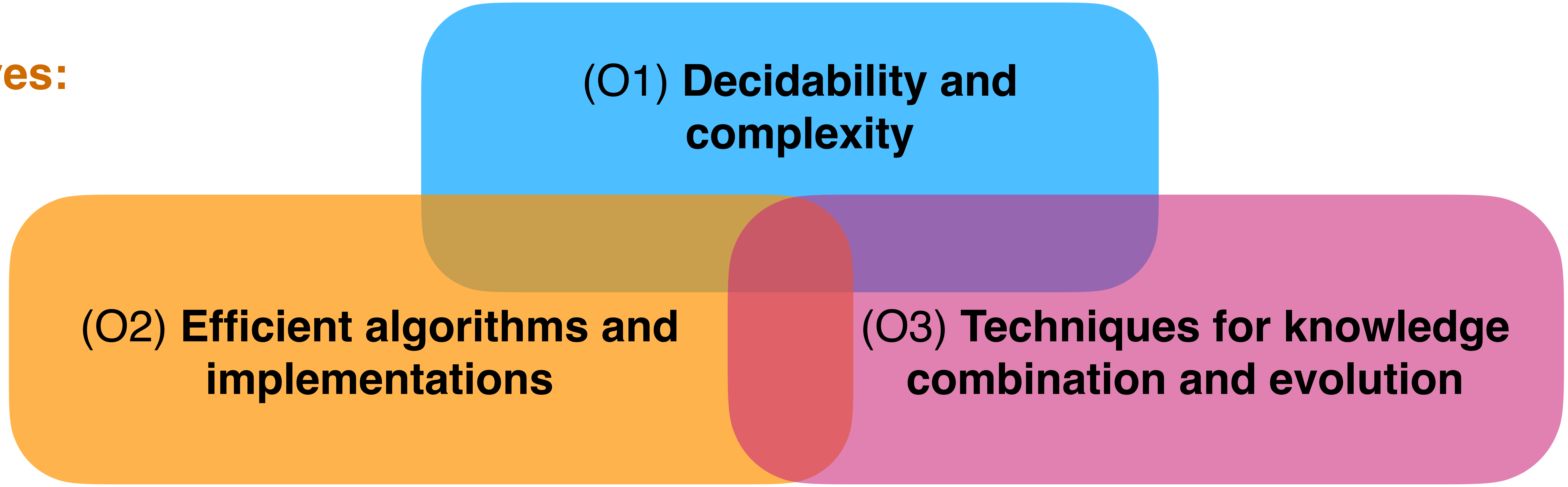
(O1) **Decidability and complexity**

(O2) **Efficient algorithms and implementations**

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

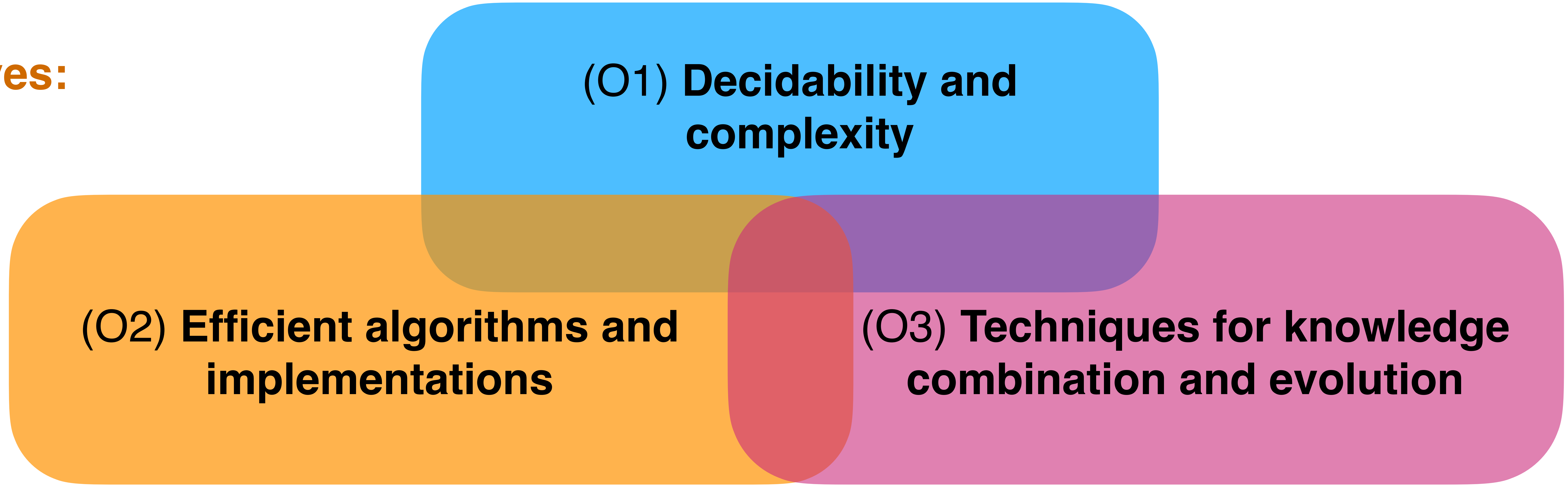
Objectives:



Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:

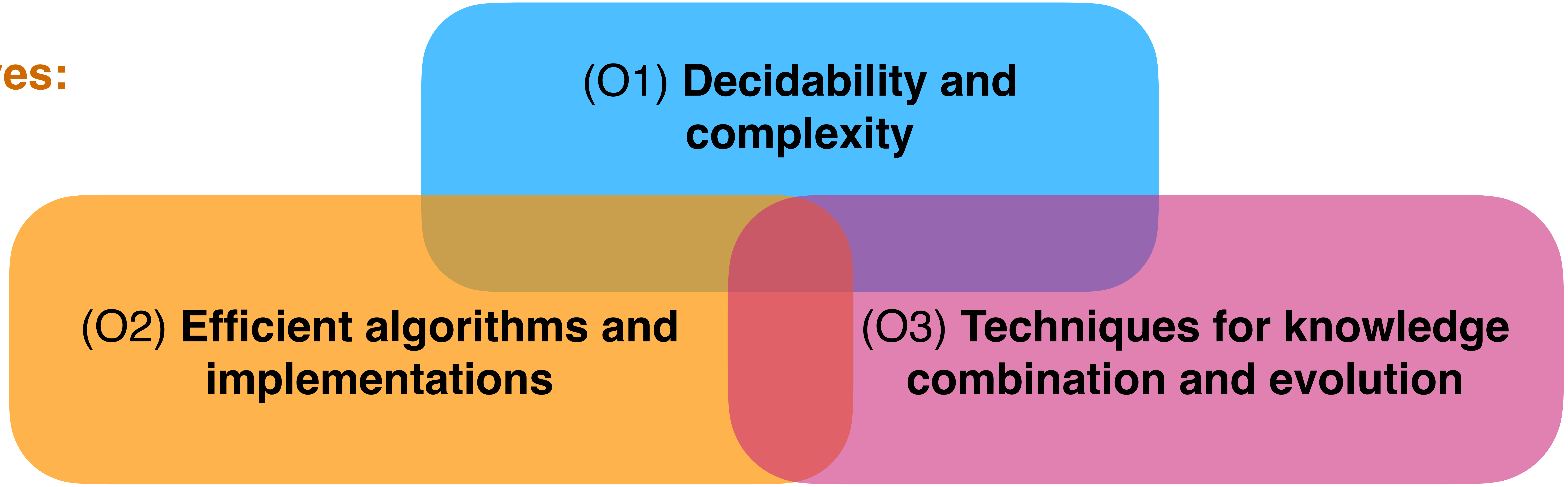


Challenge: Expressivity — Efficiency trade-off

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:



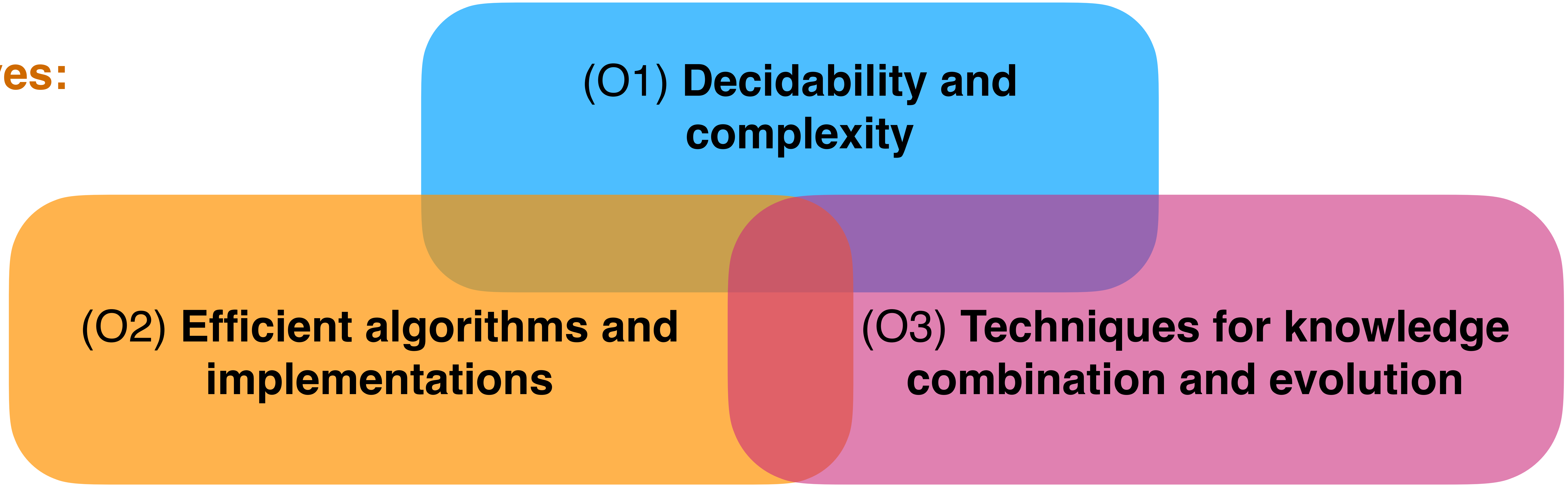
Challenge: Expressivity — Efficiency trade-off

* Knowledge available is highly diverse

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:



Challenge: Expressivity — Efficiency trade-off

- Knowledge available is highly diverse
- Multi-perspective frameworks give rise to complex reasoning tasks

Future Research Goals and Challenges

Goal: towards a viable framework for reasoning with heterogeneous knowledge communities

Objectives:

- (O1) **Decidability and complexity**
- (O2) **Efficient algorithms and implementations**
- (O3) **Techniques for knowledge combination and evolution**

Challenge: Expressivity — Efficiency trade-off

- ⋆ Knowledge available is highly diverse
- ⋆ Multi-perspective frameworks give rise to complex reasoning tasks
- ⋆ The Semantic Web contains extremely large knowledge sources

SNOMED CT

12×10^5 terms