On belief update Some novel insights

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Agenda

- In this presentation, we will showcase recent research that revolves around the KM-update model of belief change.

1. The interconnection between KM update and AGM revision.
2. The iteration of update.
3. KM-update assumes that any situation can be updated into one satisfying that input, which is unrealistic. We propose and characterize a model where not all the inputs are "reachable".
4. The model's efficacy in accurately capturing changes occurring in the world.

Introduction

AGM MODEL

Belief Revision: An example

(Gärdenfors & Rott 1995)

Beliefs:

- The bird caught in the trap is a swan
- The bird caught in the trap comes from Sweden
- Sweden is part of Europe
- All European swans are white

Consequences:

- The bird caught in the trap is white

New information:

- The bird caught in the trap is black

Which sentence(s) would you give up?

Belief Revision: An example

Some Conclusions:

- Consistency
- Minimal Change
- Logic is not enough to make a decision

AGM Model

Belief Set: Set of sentences closed under logical consequence Cn.

Cn satisfies:

inclusion ($X \subseteq Cn(X)$)
idempotence ($Cn(Cn(X)) = Cn(X)$)
monotony ($Cn(X) \subseteq Cn(Y)$ if $X \subseteq Y$)

as well as supraclassicality, deduction and compactness.

Consequently for every theory K we have that: $Cn(K) = K$.

AGM Model

Belief Set: Set of sentences closed under logical consequence Cn.

If the language is finite, we can identify K with a formula φ, such that $Cn(\varphi) = K$.

(we will assume a finite language for the rest of the talk)

AGM Model

Expansion: This operation is in charge of incorporating sentences in the original set, without eliminating any sentence from it. It allows the passage from an epistemic state in which a belief is undetermined to another epistemic state in which the belief is accepted or rejected.

Expansion is defined as $\varphi + \alpha = \varphi \wedge \alpha$.

Contraction: This operation eliminates sentences from the original set without incorporating any new ones. It allows the passage from an epistemic state in which a belief is accepted or rejected to another epistemic state in which the belief is undetermined.

Revision: This operation incorporates a sentence in the original set, but it can eliminate some beliefs in order to preserve consistency of the revised set. It allows the passage from an epistemic state in which a belief is accepted (rejected) to another state in which the belief is rejected (accepted).

Contraction vs Revision

Levi's Identity: $\varphi{*}\alpha = \varphi{-}\neg\alpha \wedge \alpha$.

Harper's Identity: $\varphi{-}\alpha = \varphi \vee \varphi{*}\neg\alpha$.

Postulates for Revision

(R1) $\varphi * \alpha \vdash \alpha$

(R2) If $\varphi \wedge \alpha \nvdash \bot$ then $\varphi * \alpha \equiv \varphi \wedge \alpha$.

(R3) If $\alpha \nvdash \bot$ then $\varphi * \alpha \nvdash \bot$.

(R4) If $\varphi_1 \equiv \varphi_2$ and $\alpha_1 \equiv \alpha_2$ then $\varphi_1 * \alpha_1 \equiv \varphi_2 * \alpha_2$.

(R5) $(\varphi * \alpha) \wedge \beta \vdash \varphi * (\alpha \wedge \beta)$

(R6) If $(\varphi * \alpha) \wedge \beta \nvdash \bot$ then $\varphi * (\alpha \wedge \beta) \vdash (\varphi * \alpha) \wedge \beta$.

Semantics: Possible Worlds

$$\|\varphi + \alpha\| = \|\varphi\| \cap \|\alpha\|$$

$$\|\varphi - \alpha\| = \|\varphi\| \cup f(\|\neg\alpha\|)$$

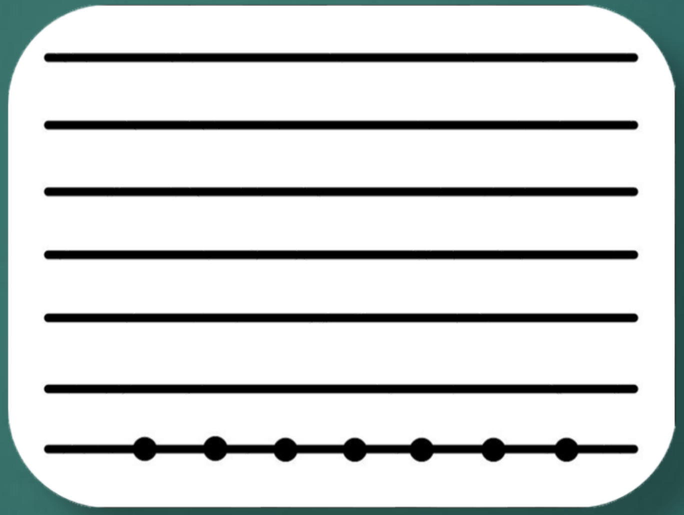
$$\|\varphi * \alpha\| = f(\|\alpha\|)$$

where $f(\|\alpha\|)$ selects a subset of $\|\alpha\|$.

Semantic: Faithful Assignment

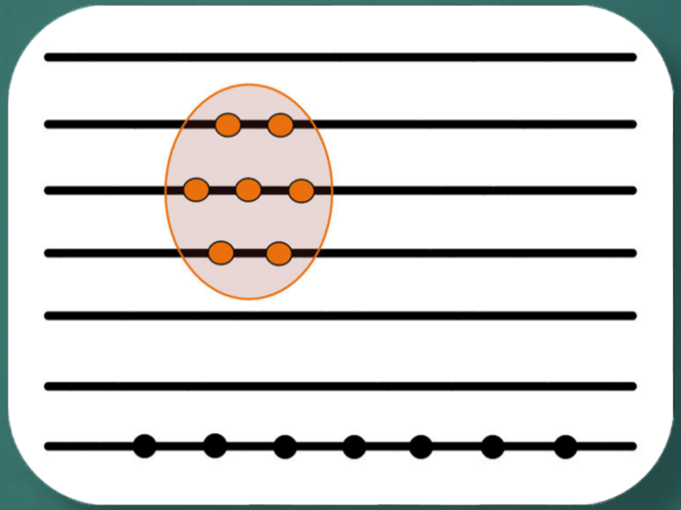
Konieczny and Pino Perez’s Notation

Semantic: Faithful Assignment



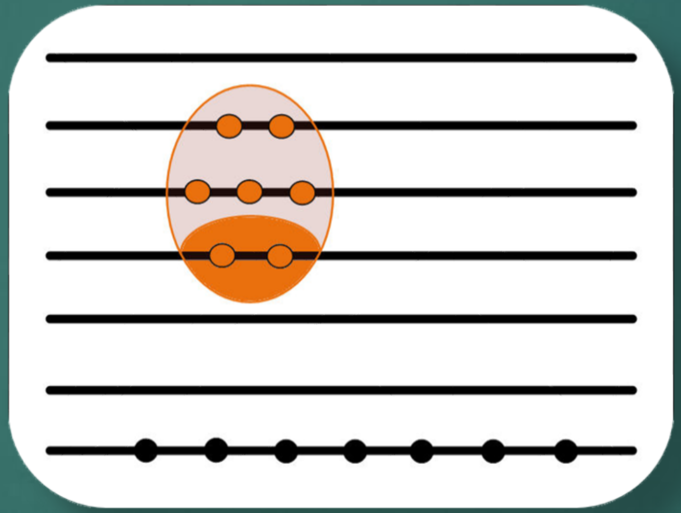
Konieczny and Pino Perez’s Notation

Semantic: Faithful Assignment



Konieczny and Pino Perez's Notation

Semantic: Faithful Assignment



Konieczny and Pino Perez’s Notation

Semantic: Faithful Assignment

Definition [KM91]
Let W be the set of all worlds (or interpretations) of a propositional language $\mathcal{L}$. A function that maps each sentence φ in $\mathcal{L}$ to a total preorder $\leq_\varphi$ on worlds W is called a faithful assignment if and only if:

(1) $\omega_1, \omega_2 \models \varphi$ only if $\omega_1 =_\varphi \omega_2$.
(2) $\omega_1 \models \varphi$ and $\omega_2 \not\models \varphi$ only if $\omega_1 <_\varphi \omega_2$.
(3) $\varphi \equiv \phi$ only if $\leq_\varphi = \leq_\phi$.

Semantic: Faithful Assignment

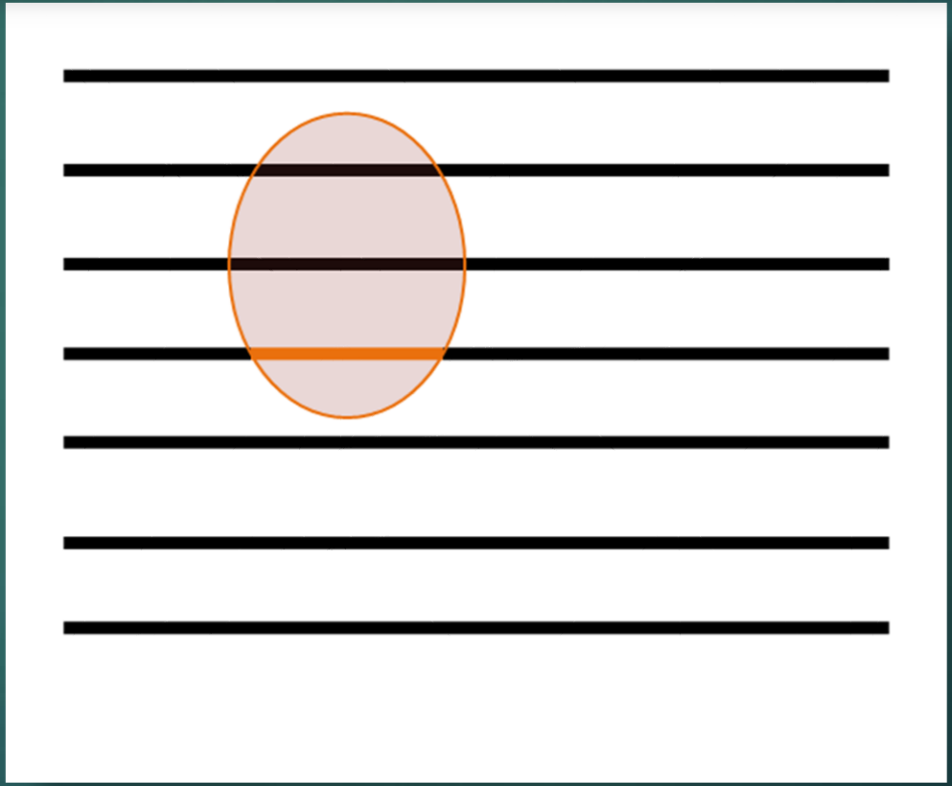
An operator $*$ is a revision operator that satisfies (R1)-(R6) if and only if there exists a faithful assignment that maps each base φ to a total pre-order $\leq_\varphi$ such that

$$\mathrm{mod}\ \varphi * \alpha = \min(\mathrm{mod}(\alpha), \leq_\varphi)$$

Iteration

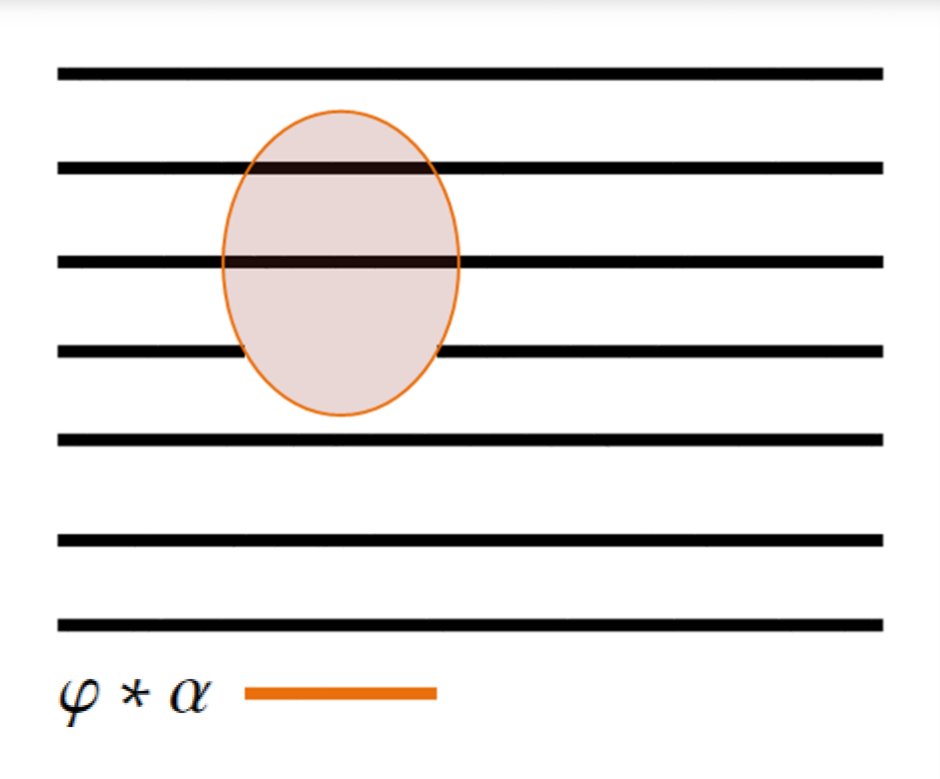
Iteration

An AGM contraction or revision takes us from a belief set to a new belief set.



Iteration

An AGM contraction or revision takes us from a belief set to a new belief set.



Iteration

However, it does not provide a new selection mechanism to be used for further changes of the new belief set.

$\varphi * \alpha$

Iteration

However, it does not provide a new selection mechanism to be used for further changes of the new belief set.

$\varphi * \alpha$

Iteration

The problem of constructing models that allow for iterated change is probably the most studied problem in the literature on belief change.

$\varphi * \alpha$

Belief States

Furthermore, the operation of change has to yield a complete such belief state representation as its outcome, not merely a new belief set.

There are several ways to represent such an extended epistemic state. The most common of these is a preorder on the set of possible worlds, or equivalently a complete sphere system.

An operation of change gives rise to a new preorder (sphere system), from which the new belief set can be inferred, and which can in its turn be subject to further changes.

AGM Revision Postulates for Belief States

(R*1) $B(\Psi * \alpha) \vdash \alpha$

(R*2) If $B(\Psi) \wedge \alpha \nvdash \bot$ then $B(\Psi * \alpha) \equiv B(\Psi) \wedge \alpha$.

(R*3) If $\alpha \nvdash \bot$ then $B(\Psi * \alpha) \nvdash \bot$.

(R*4) If $\Psi_1 = \Psi_2$ and $\alpha_1 \equiv \alpha_2$ then $B(\Psi_1 * \alpha_1) \equiv B(\Psi_2 * \alpha_2)$.

(R*5) $B(\Psi * \alpha) \wedge \beta \vdash B(\Psi * (\alpha \wedge \beta))$

(R*6) If $B(\Psi * \alpha) \wedge \beta \nvdash \bot$ then $B(\Psi * (\alpha \wedge \beta)) \vdash B(\Psi * \alpha) \wedge \beta$.

DP Postulates

(C1) If $\alpha \vdash \mu$ then $B((\Psi * \mu) * \alpha) \equiv B(\Psi * \alpha)$.
(C2) If $\alpha \vdash \neg\mu$, then $B((\Psi * \mu) * \alpha) \equiv B(\Psi * \alpha)$.
(C3) If $B(\Psi * \alpha) \vdash \mu$, then $B((\Psi * \mu) * \alpha) \vdash \mu$.
(C4) If $B(\Psi * \alpha) \nvdash \neg\mu$, then $B((\Psi * \mu) * \alpha) \nvdash \neg\mu$.

(CR1) *If* $\omega_1 \models \mu$ *and* $\omega_2 \models \mu$, *then* $\omega_1 \leqslant_{\Psi} \omega_2$ *iff* $\omega_1 \leqslant_{\Psi * \mu} \omega_2$.
(CR2) *If* $\omega_1 \models \neg\mu$ *and* $\omega_2 \models \neg\mu$, *then* $\omega_1 \leqslant_{\Psi} \omega_2$ *iff* $\omega_1 \leqslant_{\Psi * \mu} \omega_2$.
(CR3) *If* $\omega_1 \models \mu$ *and* $\omega_2 \models \neg\mu$, *then* $\omega_1 <_{\Psi} \omega_2$ *only if* $\omega_1 <_{\Psi * \mu} \omega_2$.
(CR4) *If* $\omega_1 \models \mu$ *and* $\omega_2 \models \neg\mu$, *then* $\omega_1 \leqslant_{\Psi} \omega_2$ *only if* $\omega_1 \leqslant_{\Psi * \mu} \omega_2$.

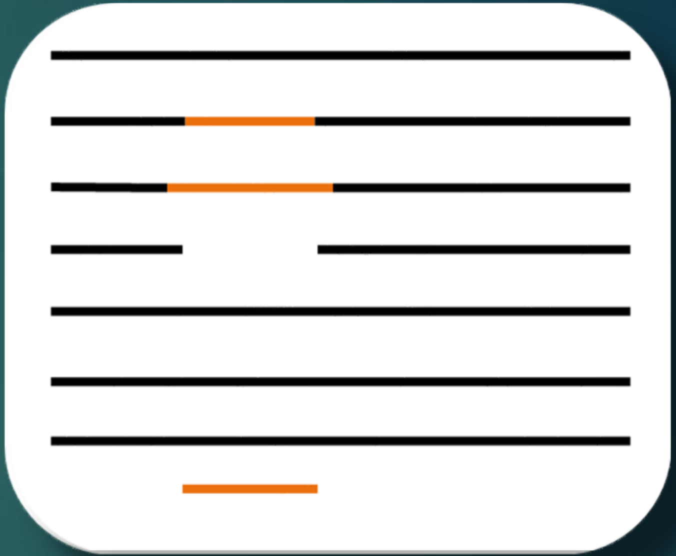
Conservative Revision

Conservative revision, originally called natural revision, has been studied by Boutilier. This operation is conservative in the sense that it only makes the minimal changes of the preorder that are needed to accept the input.

In revision by α, the maximal α-worlds are moved to the top of the preorder which is otherwise left unchanged.

(Nat*) If $B(\Psi * \alpha) \vdash \neg\beta$, then $B((\Psi * \alpha) * \beta) = B(\Psi * \beta)$.

(NatR*) If $B(\Psi * \alpha) \nsubseteq \omega_1$ and $B(\Psi * \alpha) \nsubseteq \omega_2$, then $\omega_1 \leq_\Psi \omega_2$ iff $\omega_1 \leq_{\Psi * \alpha} \omega_2$



Lexicographic Revision

Moderate revision, also called lexicographic revision, was originally studied by Nayak. When revising by α it rearranges the preorder by putting the α-worlds at top (but conserving their relative order) and the $\neg\alpha$-worlds at bottom (but conserving their relative order).

(Lex*) If $\beta \nvdash \neg\alpha$, then $B((\Psi * \alpha) * \beta) \vdash \alpha$.

(LexR*) If $\alpha \in \omega_1$ and $\neg\alpha \in \omega_2$, then $\omega_1 <_{\Psi * \alpha} \omega_2$

UPDATE

Update

In 1992, Katsuno and Mendelzon presented a type of operator of change that they called update. Whereas revision operators are intended to capture the change yielded by evolving knowledge about a static situation, update operators are intrended to mirror the change in knowledge produced by an evolving situation.

- Update is the process that allows to adapt our beliefs to some transformations that occurred in the world.
- Distinct from Belief Revision that is the process to correct some of our erroneous beliefs about the world.

Update

- Belief revision is a selection process
- Update is more than a localized revision
- Update is a transition process

Update

$$\varphi = t \wedge e \quad \vee \quad \neg t \wedge \neg e$$

Thanks Sébastien Konieczny for the example!

Update

$$\varphi = t \wedge e \quad \vee \quad \neg t \wedge \neg e$$

$$\mu = t$$

Thanks Sébastien Konieczny for the example!

Update

$\varphi = t \wedge e \quad \vee \quad \neg t \wedge \neg e$

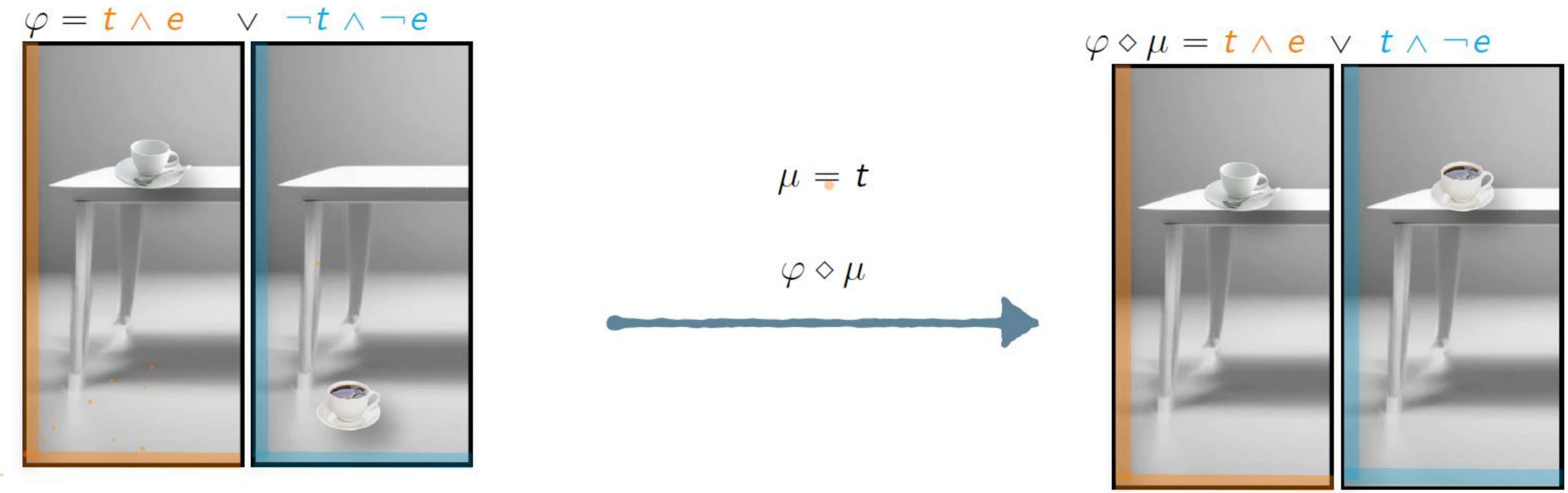
$\mu = t$

$\varphi \circ \mu$

$\varphi \circ \mu = t \wedge e$

Thanks Sébastien Konieczny for the example!

Update



Thanks Sébastien Konieczny for the example!

Update

(U1) $\varphi \diamond \alpha \vdash \alpha$

(U2) *If* $\varphi \vdash \alpha$, *then* $\varphi \diamond \alpha \equiv \varphi$.

(U3) *If* $\varphi \nvdash \bot$ *and* $\alpha \nvdash \bot$, *then* $\varphi \diamond \alpha \nvdash \bot$.

(U4) *If* $\varphi_1 \equiv \varphi_2$ *and* $\alpha_1 \equiv \alpha_2$ *then* $\varphi_1 \diamond \alpha_1 \equiv \varphi_2 \diamond \alpha_2$.

(U5) $(\varphi \diamond \alpha) \wedge \beta \vdash \varphi \diamond (\alpha \wedge \beta)$

(U6) *If* $\varphi \diamond \alpha \vdash \beta$ *and* $\varphi \diamond \beta \vdash \alpha$, *then* $\varphi \diamond \alpha \equiv \varphi \diamond \beta$.

(U7) *If* φ *is a complete formula, then* $(\varphi \diamond \alpha) \wedge (\varphi \diamond \beta) \vdash \varphi \diamond (\alpha \vee \beta)$.

(U8) $(\varphi \vee \phi) \diamond \alpha \equiv (\varphi \diamond \alpha) \vee (\phi \diamond \alpha)$

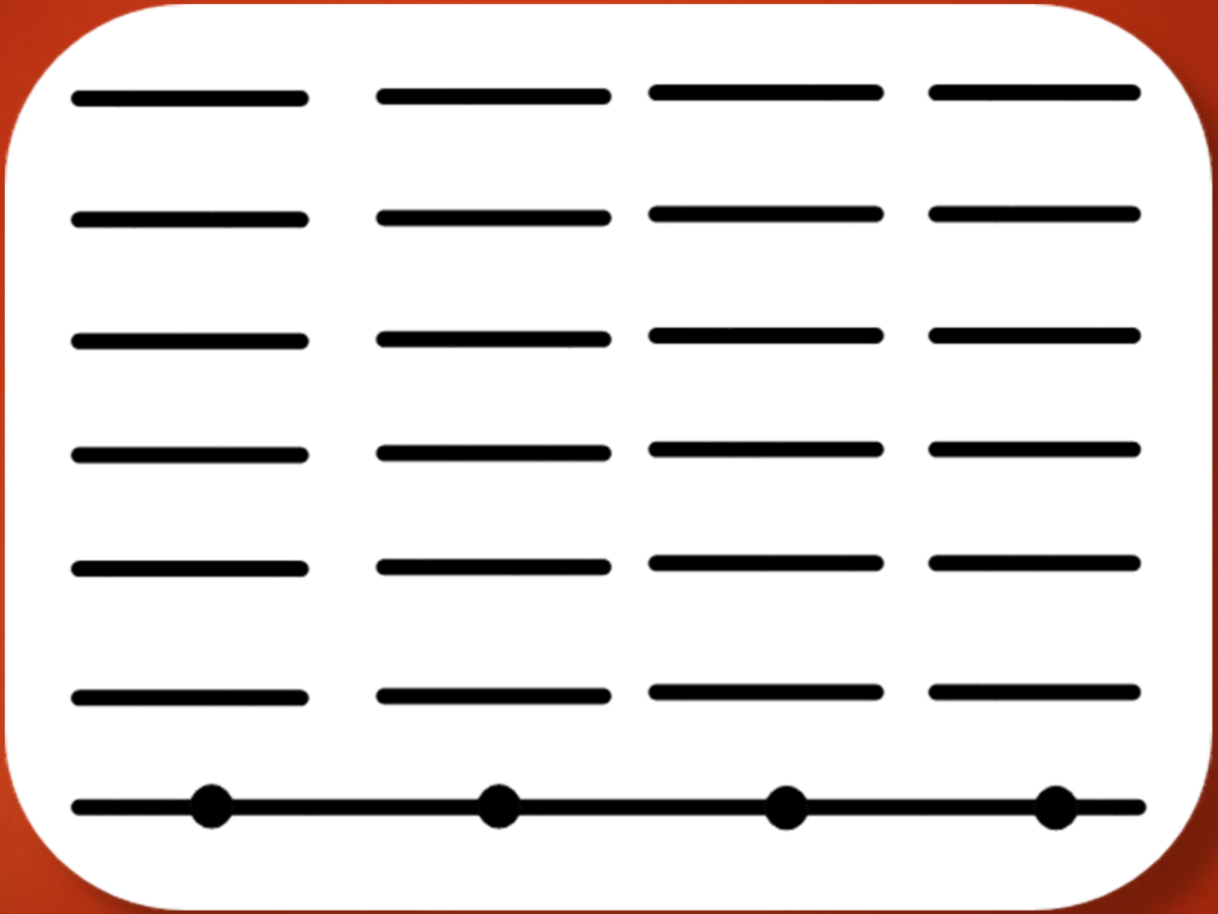
(U9) *If* φ *is a complete formula and* $(\varphi \diamond \alpha) \wedge \beta$ *is satisfiable, then* $\varphi \diamond (\alpha \wedge \beta) \vdash (\varphi \diamond \alpha) \wedge \beta$

Update

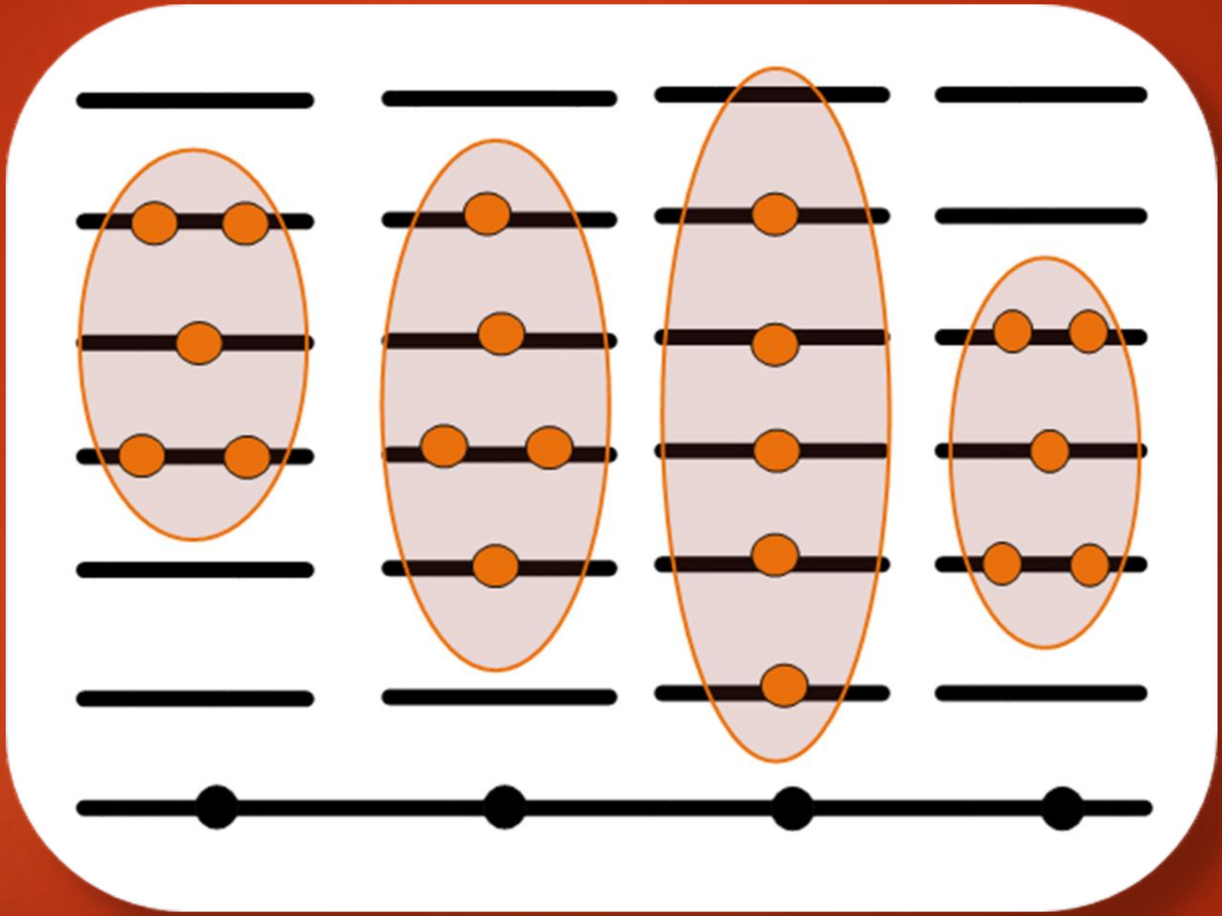
An update operator $\diamond$ satisfies (U1)-(U8) if and only if there exists a faithful assignment that maps each interpretation ω to a partial pre-order $\leq_\omega$ and such that

$$\text{mod } \varphi \diamond \mu = \bigcup_{\omega \models \varphi} \min(\text{mod } \mu, \leq_\omega)$$

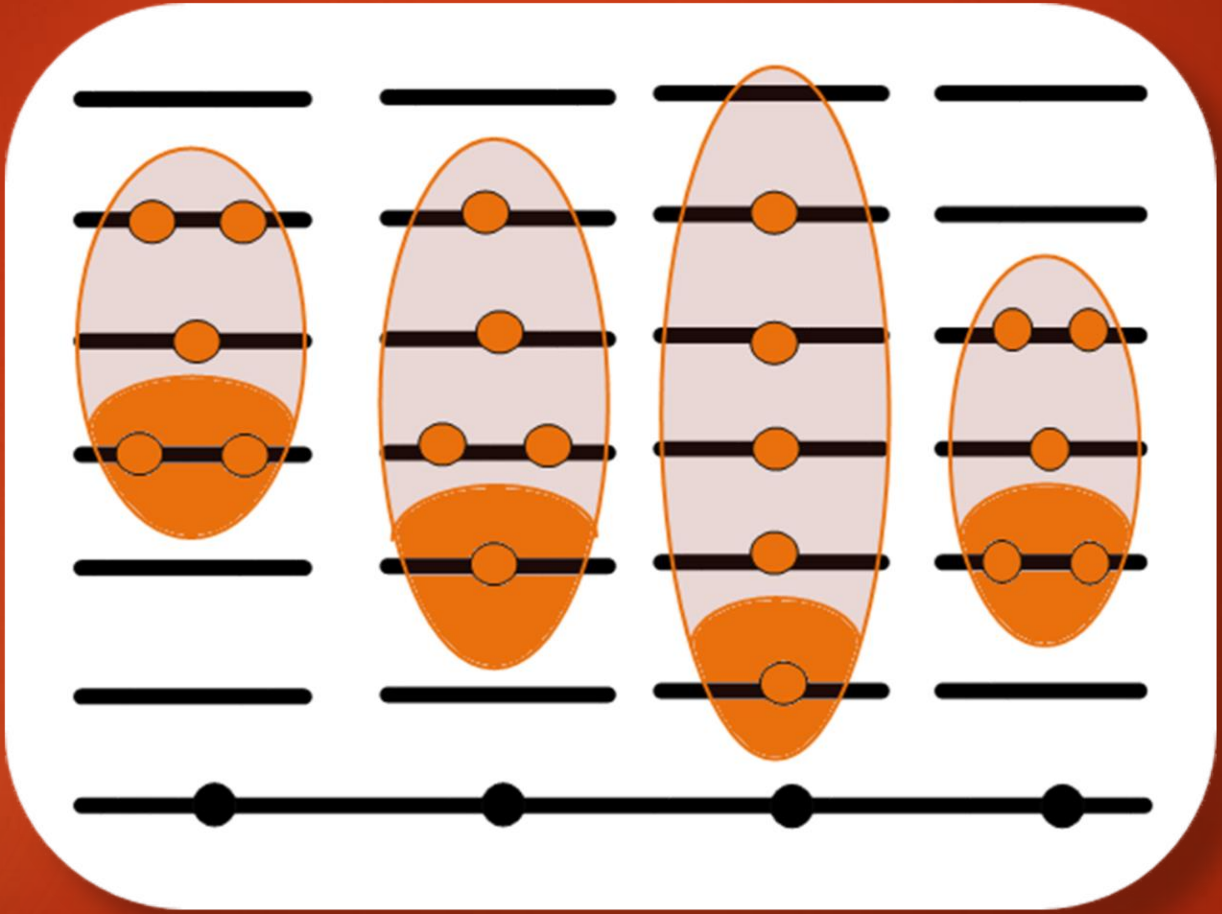
Update



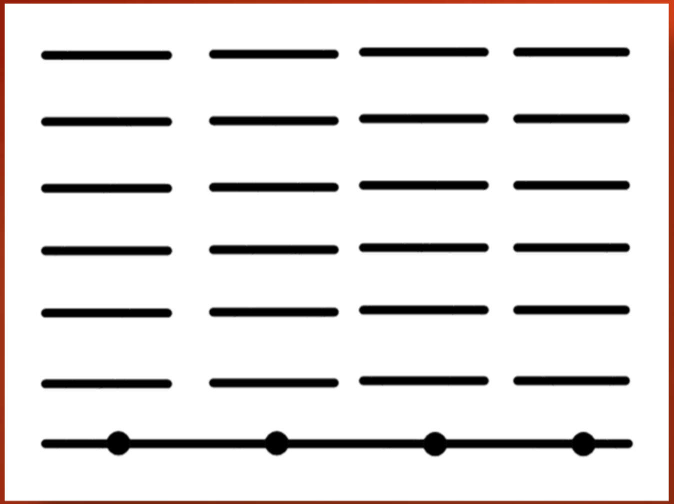
Update

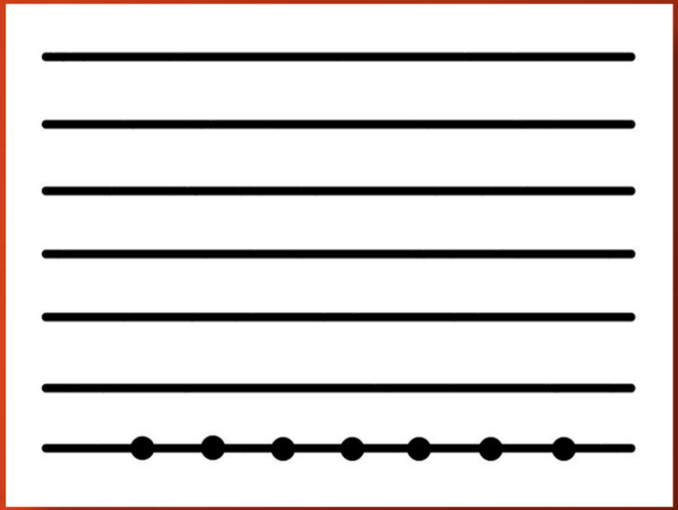


Update

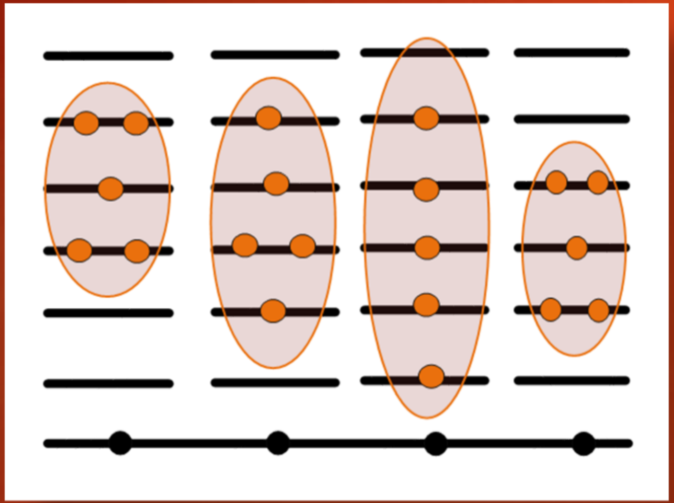


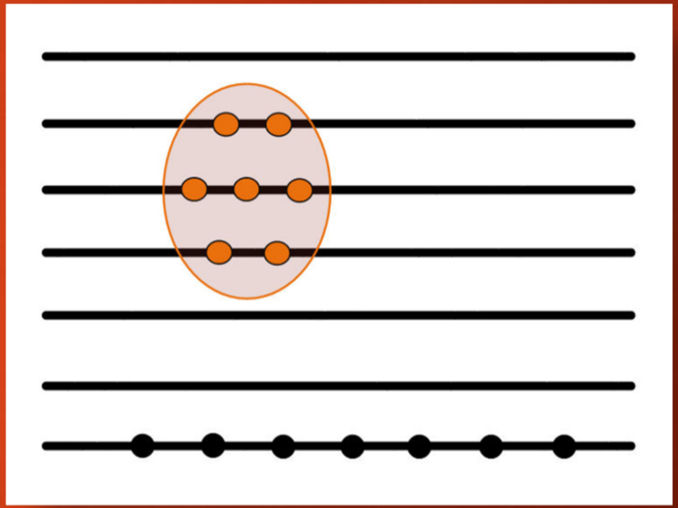
Update vs Revision



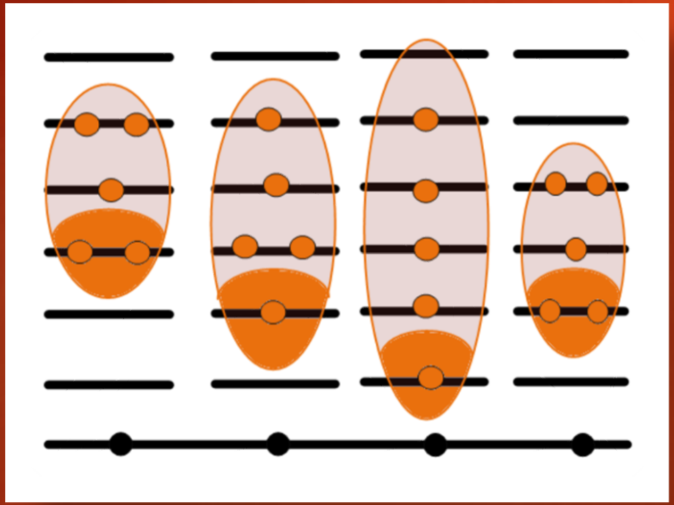


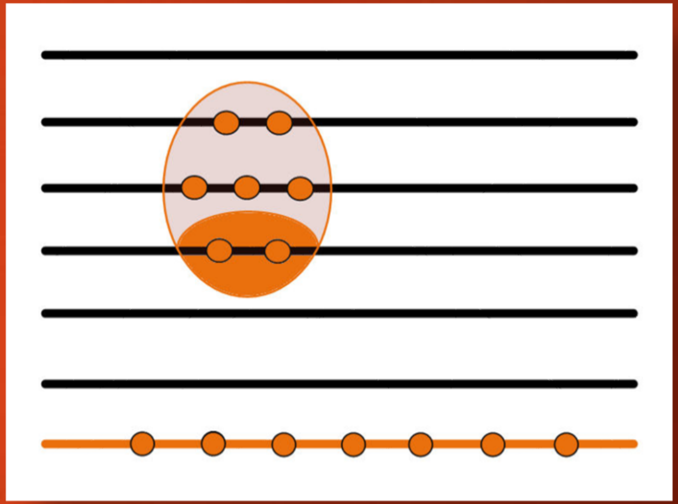
Update vs Revision





Update vs Revision





Update vs Revision

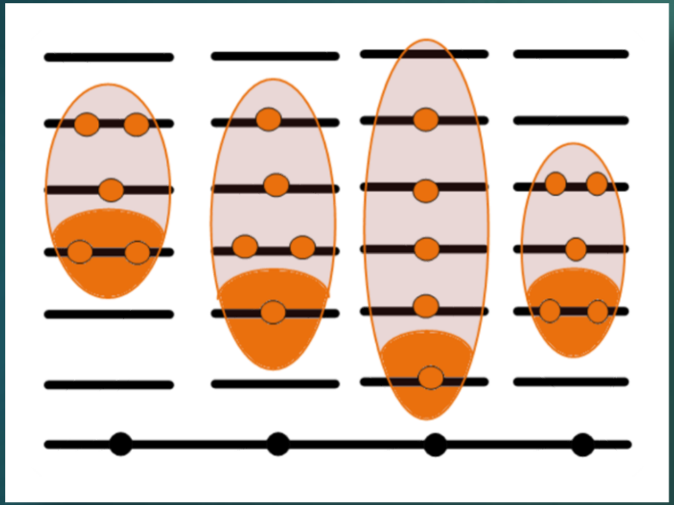
(U2) *If* $\varphi \vdash \alpha$, *then* $\varphi \diamond \alpha \equiv \varphi$.

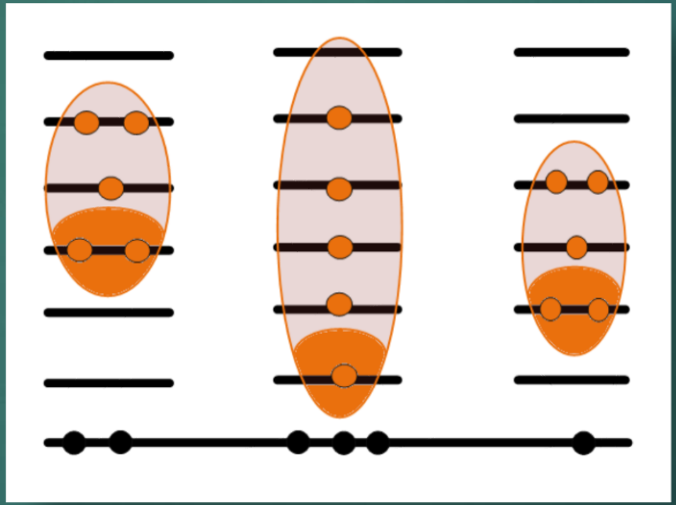
(R2) If $\varphi \wedge \alpha \nvdash \bot$ then $\varphi * \alpha \equiv \varphi \wedge \alpha$.

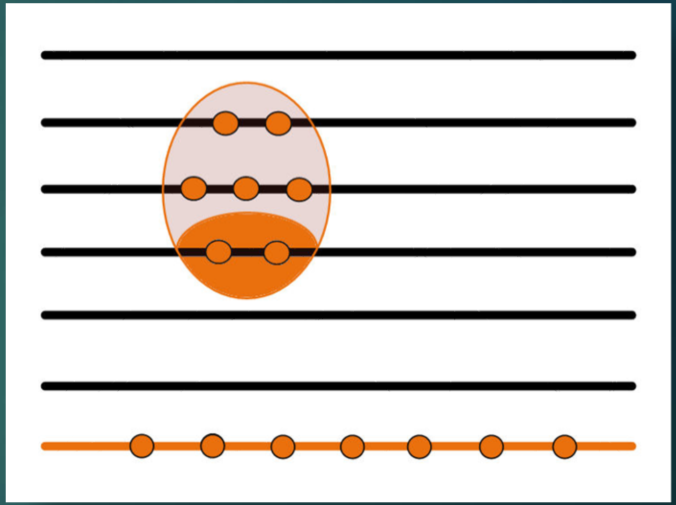
Update and Revision Unified

BASED ON AN ONGOING WORK WITH GABRIELE KERN ISBERNER, TOMMIE MEYER AND ABHAYA NAYAK

Update and Revision Unified







Update and Revision Unified

In order to represent the partition of $\|\varphi\|$ syntactically, we will use a function s such that

$$
\begin{aligned}
s(\varphi) = \{\varphi_1 \ldots \varphi_n\} \text{ such that}\\
&\varphi_i \vdash \varphi,\\
&\varphi_i \nvdash \bot,\\
&\varphi_i \wedge \varphi_j \vdash \bot \text{ for } i \neq j \text{ and}\\
&\textstyle\bigvee_{i=1}^{n} \varphi_i \equiv \varphi.
\end{aligned}
$$

Update and Revision Unified

(Un1) $\varphi \circledast \alpha \vdash \alpha$

(Un2) If $\varphi \vdash \alpha$, then $\varphi \circledast \alpha \equiv \varphi$.

(Un2') If $s(\varphi) = \{\varphi\}$ and $\varphi \wedge \alpha \nvdash \bot$ then $\varphi \circledast \alpha \equiv \varphi \wedge \alpha$.

(Un3) If $\varphi \nvdash \bot$ and $\alpha \nvdash \bot$, then $\varphi \circledast \alpha \nvdash \bot$.

(Un4) If $\varphi_1 \equiv \varphi_2$ and $\alpha_1 \equiv \alpha_2$ then $\varphi_1 \diamond \alpha_1 \equiv \varphi_2 \circledast \alpha_2$.

(Un5) $(\varphi \circledast \alpha) \wedge \beta \vdash \varphi \circledast (\alpha \wedge \beta)$

(Un6) If $\varphi \circledast \alpha \vdash \beta$ and $\varphi \circledast \beta \vdash \alpha$, then $\varphi \circledast \alpha \equiv \varphi \circledast \beta$.

(Un7) If $s(\varphi) = \{\varphi\}$, then $(\varphi \circledast \alpha) \wedge (\varphi \circledast \beta) \vdash \varphi \circledast (\alpha \vee \beta)$.

(Un8) $\varphi \diamond \alpha \equiv \bigvee_{i=1}^{n} \varphi_i \circledast \alpha$

(Un9) If $s(\varphi) = \{\varphi\}$ and $(\varphi \circledast \alpha) \wedge \beta$ is satisfiable,
then $\varphi \circledast (\alpha \wedge \beta) \vdash (\varphi \circledast \alpha) \wedge \beta$

Update and Revision Unified

Theorems:

An operator $\circledast$ satisfied conditions (Un1) – (Un8) if and only if there exists a faithful assignment that maps $s(\varphi)$ to a preorders $(\leqslant_{\varphi_1}, \ldots, \leqslant_{\varphi_n})$ such that

$$\|\varphi \circledast \alpha\| = \bigcup_{i=1\ldots n} \min(\|\alpha\|, \leqslant_{\varphi_i})$$

An operator $\circledast$ satisfied conditions (Un1) – (Un9) if and only if there exists a faithful assignment that maps $s(\varphi)$ to a total preorders $(\leqslant_{\varphi_1}, \ldots, \leqslant_{\varphi_n})$ such that

$$\|\varphi \circledast \alpha\| = \bigcup_{i=1\ldots n} \min(\|\alpha\|, \leqslant_{\varphi_i})$$

Iteration of Update

BASED ON "ON THE LOGIC OF THEORY CHANGE: ITERATION OF KM-UPDATE"

EDUARDO FERMÉ – SARA GONÇALVES.

Key Idea

KM-update operators have been defined for all potential belief sets φ.

At first glance, iteration does not necessitate special consideration as $(\varphi \diamond \alpha) \diamond \beta$ is well-defined.

Key Idea

Observation Let $\diamond$ be an update operator. Then $\diamond$ satisfies:

(CU1') If $\alpha \vdash \mu$ then $(\varphi \diamond \mu) \diamond \alpha \equiv \varphi \diamond \alpha$
(CU2') If $\alpha \vdash \neg\mu$, then $(\varphi \diamond \mu) \diamond \alpha \equiv \varphi \diamond \alpha$
(CU3') If $\varphi \diamond \alpha \vdash \mu$, then $(\varphi \diamond \mu) \diamond \alpha \vdash \mu$
(CU4') If $\varphi \diamond \alpha \nvdash \neg\mu$, then $(\varphi \diamond \mu) \diamond \alpha \nvdash \neg\mu$

Key Idea

If changes in preferences are desired as a result of updating by α, a new operator $\blacklozenge$ must be defined to reflect these changes.

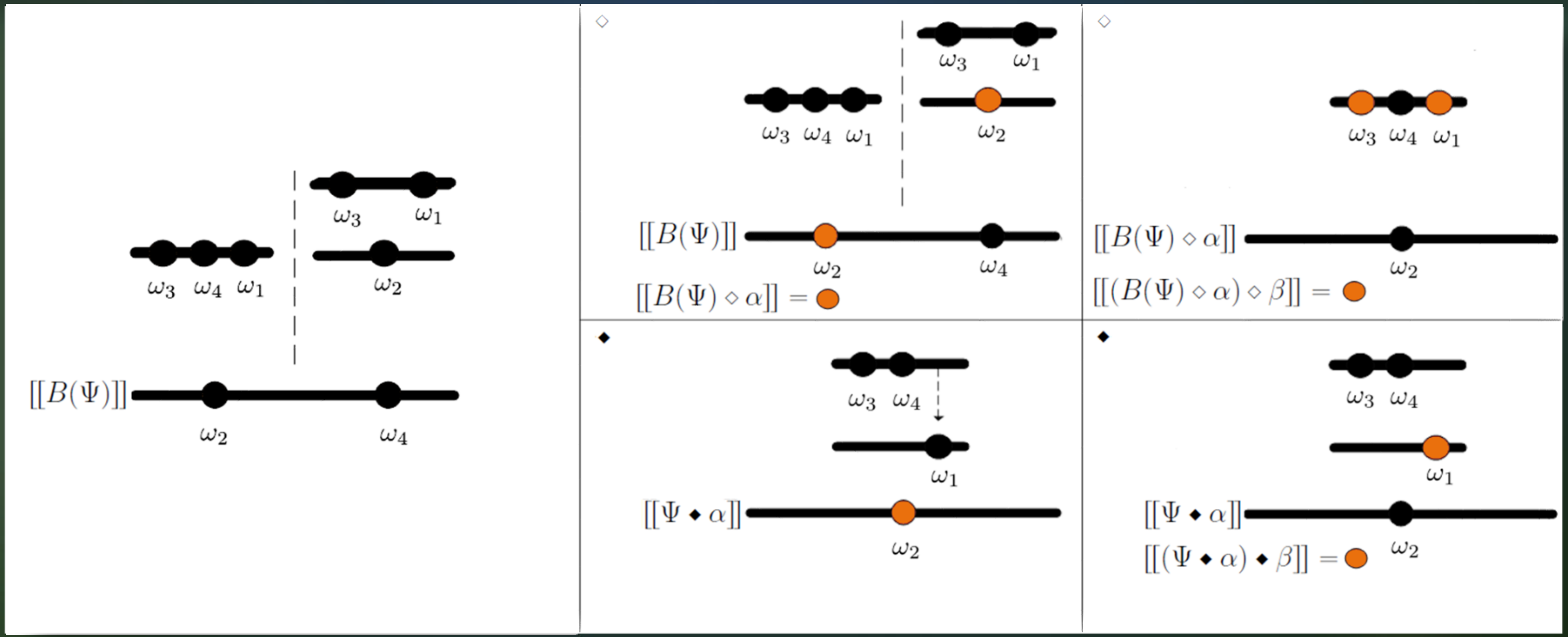
The new operator $\blacklozenge$ not only modifies the belief set φ, but also impacts the preferences for future updates.

Update vs Iterated Update

$$\|\{\alpha, \beta\}\| = \{\omega_1\}$$
$$\|\{\alpha, \neg\beta\}\| = \{\omega_2\}$$
$$\|\{\neg\alpha, \beta\}\| = \{\omega_3\}$$
$$\|\{\neg\alpha, \neg\beta\}\| = \{\omega_4\}$$

$$B(\Psi) \equiv \neg\beta$$

$$\|B(\Psi) \diamond \alpha\| = \{\omega_2\}.$$
$$\|\Psi \blacklozenge \alpha\| = \{\omega_2\}.$$
$$\|(B(\Psi) \diamond \alpha) \diamond \beta\| = \{\omega_1, \omega_3\}.$$
$$\|(\Psi \blacklozenge \alpha) \blacklozenge \beta\| = \{\omega_1\}.$$



Update operator for belief states

Definition (adapted from [DP97])

An epistemic state Ψ is a tuple $\Psi = \langle \varphi, \diamond \rangle$, where $B(\Psi) = \varphi$ is a propositional formula that denotes the current beliefs of the agent in the epistemic state Ψ and $O(\Psi) = \diamond$ is a KM-update operator.

Definition

Let φ and α sentences of $\mathcal{L}$. Let $\diamond$ be a KM-update operator. Let $\Psi = \langle \varphi, \diamond \rangle$ be a belief state. $\blacklozenge$ is an *update operator* for Ψ if and only if satisfies the following properties:

(1) $B(\Psi \blacklozenge \alpha) = \varphi \diamond \alpha$
(2) If $O(\Psi_1) = O(\Psi_2)$ then $O(\Psi_1 \blacklozenge \alpha) = O(\Psi_2 \blacklozenge \alpha)$ for all $\alpha \in \mathcal{L}$.

The second property implies that the operator $\blacklozenge$ will modify $\diamond$ independently of the belief set.

Properties of the Update Operator

(U•1) $B(\Psi \bullet \alpha) \vdash \alpha$
(U•2) If $B(\Psi) \vdash \alpha$, then $B(\Psi \bullet \alpha) \equiv B(\Psi)$.
(U•3) If $B(\Psi) \nvdash \bot$ and $\alpha \nvdash \bot$, then $B(\Psi \bullet \alpha) \nvdash \bot$.
(U•4) If $B(\Psi_1) \equiv B(\Psi_2)$, $O(\Psi_1) = O(\Psi_2)$ and $\alpha_1 \equiv \alpha_2$ then $B(\Psi_1 \bullet \alpha_1) \equiv B(\Psi_2 \bullet \alpha_2)$.
(U•5) $B(\Psi \bullet \alpha) \wedge \beta \vdash B(\Psi \bullet (\alpha \wedge \beta))$
(U•6) If $B(\Psi \bullet \alpha) \vdash \beta$ and $B(\Psi \bullet \beta) \vdash \alpha$, then $B(\Psi \bullet \alpha) \equiv B(\Psi \bullet \beta)$.
(U•7) If $B(\Psi)$ is a complete formula, then $B(\Psi \bullet \alpha) \wedge B(\Psi \bullet \beta) \vdash B(\Psi \bullet (\alpha \vee \beta))$.
(U•8) If $O(\Psi_1) = O(\Psi_2) = O(\Psi_3)$, and $B(\Psi_1) \equiv B(\Psi_2) \vee B(\Psi_3)$, then $B(\Psi_1 \bullet \alpha) \equiv B(\Psi_2 \bullet \alpha) \vee B(\Psi_3 \bullet \alpha)$.

Postulates for Iteration

(CU1) If $\alpha \vdash \mu$ then $B((\Psi \bullet \mu) \bullet \alpha) \equiv B(\Psi \bullet \alpha)$.
(CU2) If $\alpha \vdash \neg\mu$, then $B((\Psi \bullet \mu) \bullet \alpha) \equiv B(\Psi \bullet \alpha)$.
(CU3) If $B(\Psi \bullet \alpha) \vdash \mu$, then $B((\Psi \bullet \mu) \bullet \alpha) \vdash \mu$.
(CU4) If $B(\Psi \bullet \alpha) \nvdash \neg\mu$, then $B((\Psi \bullet \mu) \bullet \alpha) \nvdash \neg\mu$.

(CRU1) If $\omega_1 \models \mu$ and $\omega_2 \models \mu$, then $\omega_1 \leqslant_{\{\Psi,\omega\}} \omega_2 \Leftrightarrow \omega_1 \leqslant_{\{\Psi\bullet\mu,\omega\}} \omega_2$.
(CRU2) If $\omega_1 \models \neg\mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 \leqslant_{\{\Psi,\omega\}} \omega_2 \Leftrightarrow \omega_1 \leqslant_{\{\Psi\bullet\mu,\omega\}} \omega_2$.
(CRU3) If $\omega_1 \models \mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 <_{\{\Psi,\omega\}} \omega_2$ implies $\omega_1 <_{\{\Psi\bullet\mu,\omega\}} \omega_2$.
(CRU4) If $\omega_1 \models \mu$ and $\omega_2 \models \neg\mu$, then $\omega_1 \leqslant_{\{\Psi,\omega\}} \omega_2$ implies $\omega_1 \leqslant_{\{\Psi\bullet\mu,\omega\}} \omega_2$.

Theorem

Theorem
An operator $\diamond$ satisfies (U$\diamond$1)-(U$\diamond$8) if and only if there exists a faithful assignment that maps each possible world ω to a partial preorder $\leqslant_{\{\Psi,\omega\}}$ such that:

$$[[\Psi \diamond \alpha]] = \bigcup_{\omega \models B(\Psi)} \min([[\alpha]], \leqslant_{\{\Psi,\omega\}}).$$

Theorem
Let $\diamond$ be an update operator and let f be a faithful assignment that maps each possible world ω to a partial preorder $\leqslant_{\{\Psi,\omega\}}$. Then $\diamond$

1. satisfies (CU1) iff $\leqslant_{\{\Psi,\omega\}}$ satisfies (CRU1)
2. satisfies (CU2) iff $\leqslant_{\{\Psi,\omega\}}$ satisfies (CRU2)
3. satisfies (CU3) iff $\leqslant_{\{\Psi,\omega\}}$ satisfies (CRU3)
4. satisfies (CU4) iff $\leqslant_{\{\Psi,\omega\}}$ satisfies (CRU4)

Credible Update

BASED ON “CREDIBLE MODELS OF BELIEF UPDATE”.

EDUARDO FERMÉ, SÉBASTIEN KONIECZNY, RAMÓN PINO PÉREZ AND NICOLAS SCHWIND.

Some transitions in the world are possible

Fill the cup

Broke The cup

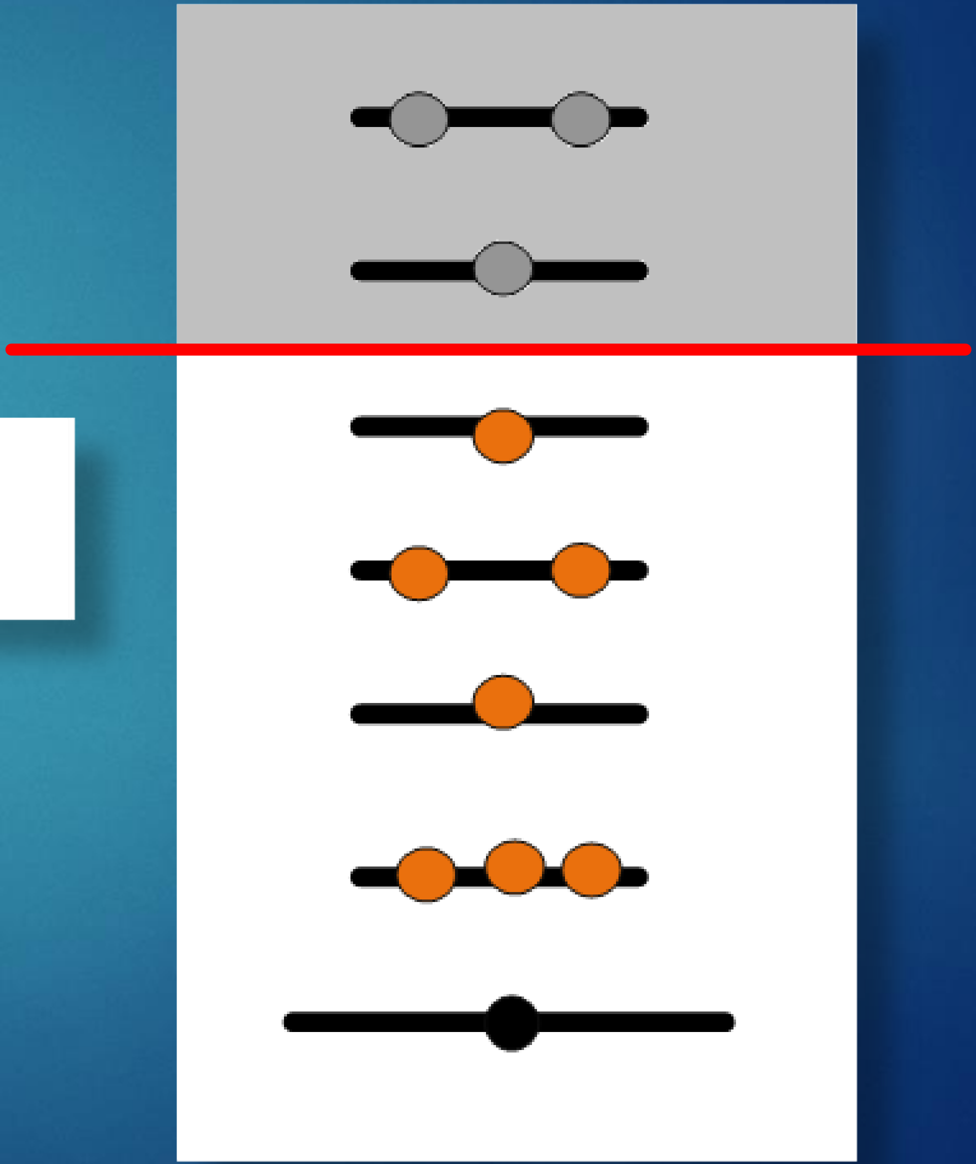
Other not

~~Fill the cup~~

~~Unbroke The cup~~

What this means?

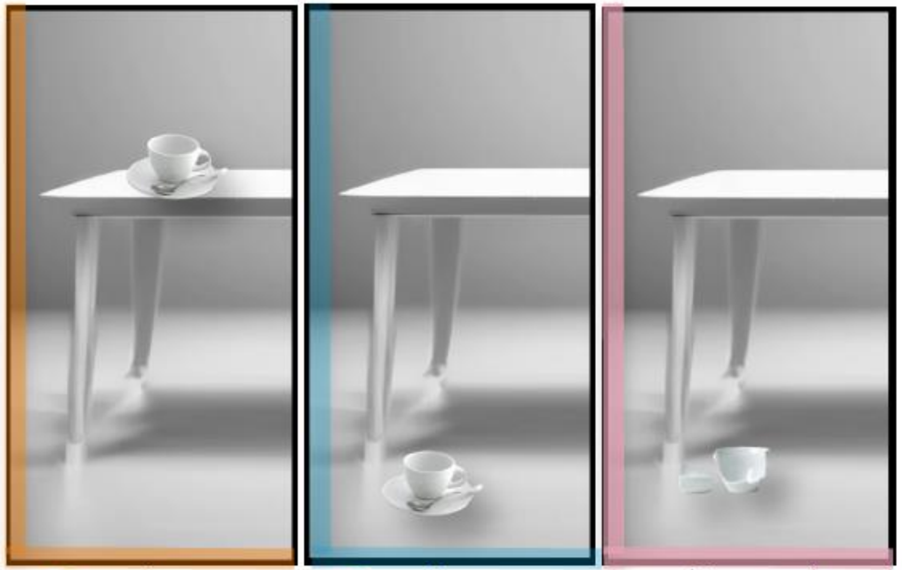
(reachability) $\forall \omega, \omega' \in \Omega \;\; \omega \leq_\omega \omega'$



What can we do?

Models: Case (a)

$\varphi = t \wedge e \wedge \neg b \vee \neg t \wedge e \wedge \neg b \vee \neg t \wedge e \wedge b$

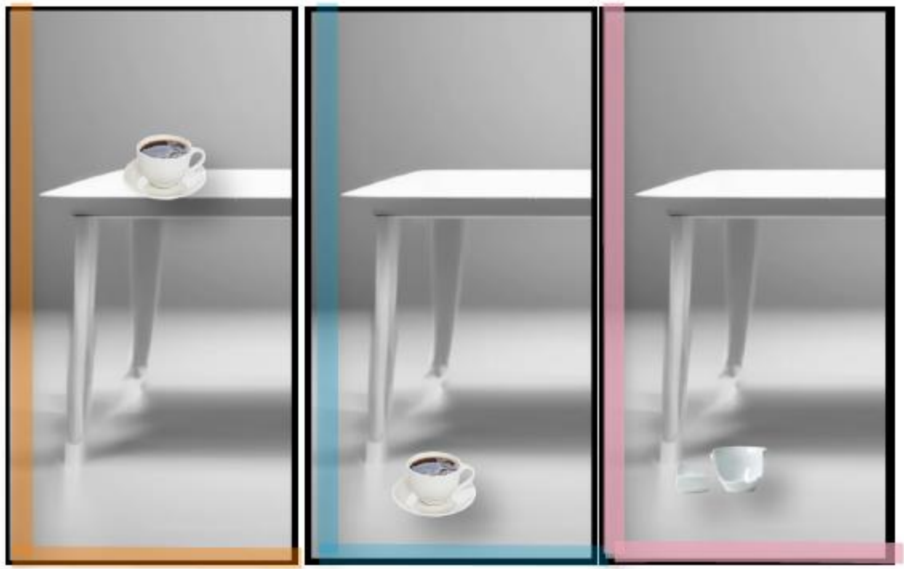


$C = b \rightarrow e$ $C = b \rightarrow e$ $C = (b \rightarrow e) \wedge b$

$\mu = \neg e$

$\varphi \diamond \mu$

$\varphi \diamond \mu = t \wedge \neg e \wedge \neg b \vee \neg t \wedge \neg e \wedge \neg b \vee \neg t \wedge e \wedge b$



Please Robbie, could you go in the other room and fill the cup ?”

Thanks (again) Sébastien Konieczny for the example!

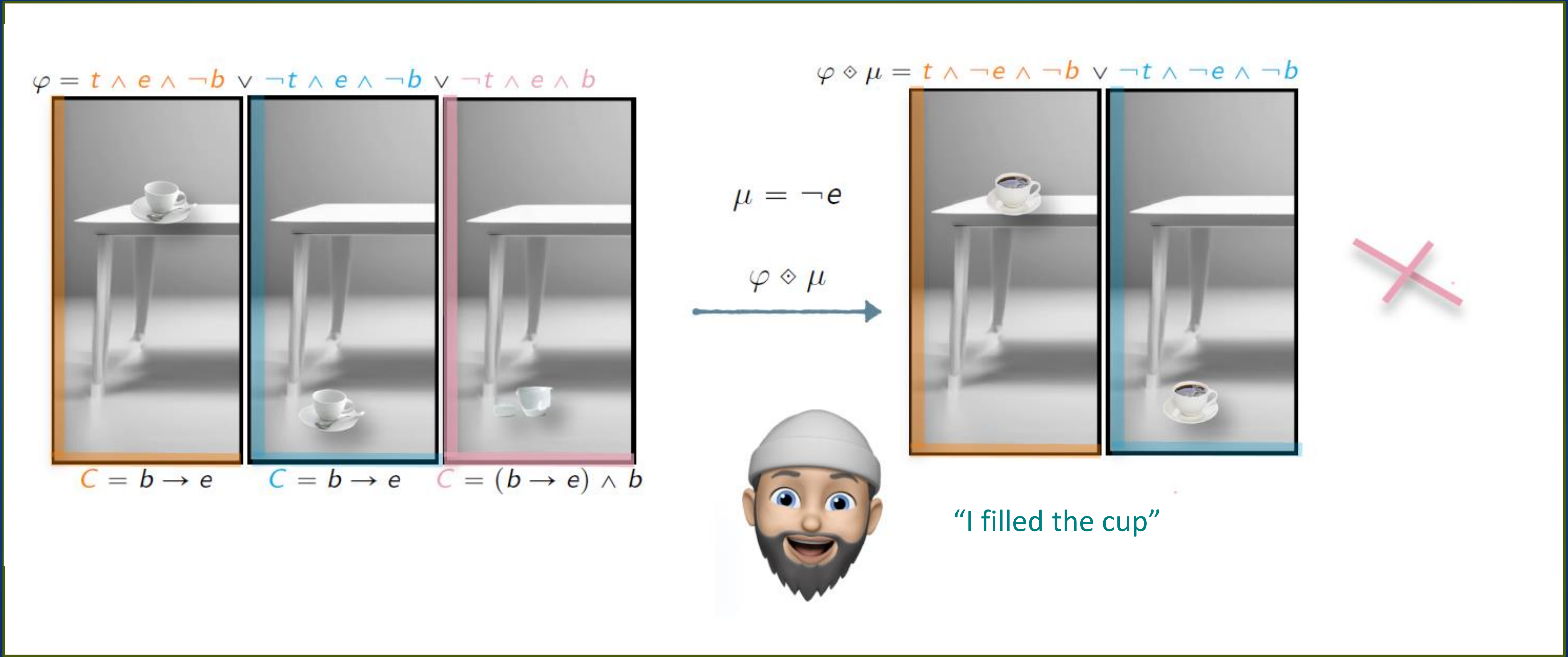
Models: Case (a)

~~**(U1)** $\varphi \diamond \alpha \vdash \alpha$~~
(U2) *If* $\varphi \vdash \alpha$, *then* $\varphi \diamond \alpha \equiv \varphi$.
(U3) *If* $\varphi \nvdash \bot$ *and* $\alpha \nvdash \bot$, *then* $\varphi \diamond \alpha \nvdash \bot$.
(U4) *If* $\varphi_1 \equiv \varphi_2$ *and* $\alpha_1 \equiv \alpha_2$ *then* $\varphi_1 \diamond \alpha_1 \equiv \varphi_2 \diamond \alpha_2$.
(U5) $(\varphi \diamond \alpha) \wedge \beta \vdash \varphi \diamond (\alpha \wedge \beta)$
(U6) *If* $\varphi \diamond \alpha \vdash \beta$ *and* $\varphi \diamond \beta \vdash \alpha$, *then* $\varphi \diamond \alpha \equiv \varphi \diamond \beta$.
(U7) *If* φ *is a complete formula, then* $(\varphi \diamond \alpha) \wedge (\varphi \diamond \beta) \vdash \varphi \diamond (\alpha \vee \beta)$.
(U8) $(\varphi \vee \phi) \diamond \alpha \equiv (\varphi \diamond \alpha) \vee (\phi \diamond \alpha)$

(RSC) If φ is complete, then $\varphi \diamond \alpha \vdash \alpha$ or $\varphi \diamond \alpha \equiv \varphi$

(SM) If $\alpha \vdash \beta$ and $\varphi \diamond \alpha \vdash \alpha$, then $\varphi \diamond \beta \vdash \beta$

Models: Case (b)



Thanks (again - again) Sébastien Konieczny for the example!

Models: Case (b)

(U1) $\varphi \diamond \alpha \vdash \alpha$

(U2) *If* $\varphi \vdash \alpha$, *then* $\varphi \diamond \alpha \equiv \varphi$.

~~**(U3)** *If* $\varphi \nvdash \bot$ *and* $\alpha \nvdash \bot$, *then* $\varphi \diamond \alpha \nvdash \bot$.~~

(U4) *If* $\varphi_1 \equiv \varphi_2$ *and* $\alpha_1 \equiv \alpha_2$ *then* $\varphi_1 \diamond \alpha_1 \equiv \varphi_2 \diamond \alpha_2$.

(U5) $(\varphi \diamond \alpha) \wedge \beta \vdash \varphi \diamond (\alpha \wedge \beta)$

(U6) *If* $\varphi \diamond \alpha \vdash \beta$ *and* $\varphi \diamond \beta \vdash \alpha$, *then* $\varphi \diamond \alpha \equiv \varphi \diamond \beta$.

(U7) *If* φ *is a complete formula, then* $(\varphi \diamond \alpha) \wedge (\varphi \diamond \beta) \vdash \varphi \diamond (\alpha \vee \beta)$.

(U8) $(\varphi \vee \phi) \diamond \alpha \equiv (\varphi \diamond \alpha) \vee (\phi \diamond \alpha)$

The Credible Level

Definition[Credible faithful assignment] A *credible faithful assignment* is a mapping associating each world $\omega_i \in \Omega$ with a pair $(\mathcal{C}_i, \leq_i)$, where $\{\omega_i\} \subseteq \mathcal{C}_i \subseteq \Omega$ and $\leq_i$ is an ordering over $\mathcal{C}_i$ such that for each $\omega \in \mathcal{C}_i$, if $\omega_i \neq \omega$, then $\omega_i <_i \omega$.

Representation Theorems

Definition[Consistent credibility-limited update operator]
An operator $\diamond$ is a *consistent credibility-limited (CCL) update operator* if it satisfies (U2)-(U8), (RSC) and (SM).

Theorem An update operator $\diamond$ is a CCL update operator if and only if there exists a credible faithful assignment $\omega_i \mapsto (\leq_i, \mathcal{C}_i)$ such that for all formulae φ, α,

$$\|\varphi \diamond \alpha\| = \bigcup_{\omega_i \in \|\varphi\|} f(\omega_i, \alpha),$$

where for each $\omega_i \in \Omega$ and each formula α, $f(\omega_i, \alpha)$ is defined as

$$f(\omega_i, \alpha) = \begin{cases} \min(\|\alpha\| \cap \mathcal{C}_i, \leq_i), & \text{if } \|\alpha\| \cap \mathcal{C}_i \neq \varnothing, \\ \{\omega_i\}, & \text{otherwise.} \end{cases}$$

Representation Theorems

Definition [Credibility-limited update operator] An operator $\diamond$ is a *credibility-limited (CL) update operator* if it satisfies (U1)-(U2), and (U4)-(U8).

Theorem An update operator $\diamond$ is a CL update operator if and only if there exists a credible faithful assignment $\omega_i \mapsto (\leq_i, \mathcal{C}_i)$ such that for all formulae φ, α,

$$\|\varphi \diamond \alpha\| = \bigcup_{\omega_i \in \|\varphi\|} \min(\|\alpha\| \cap \mathcal{C}_i, \leq_i).$$

Is KM-Update really a model for update?

Example Revisited

Example Initially the agent knows that there is either a book on the table (p) or a magazine on the table (q), but not both.

Case 1: The agent is told that there is a book on the table. She concludes that there is no magazine on the table. This is revision.

Case 2: The agent is told that subsequently a book has been put on the table. In this case she should not conclude that there is no magazine on the table. This is update.

Another (counter) Examples

Example Initially the agent knows that the wall of the room is white (p). The agent is told that subsequently that the wall of the room was repainted white (p) or grey (q) but not both.

(U2) *If* $\varphi \vdash \alpha$, *then* $\varphi \diamond \alpha \equiv \varphi$.

Another (counter) example

Example My initial belief is that either Alice (a) is in the office or Bob (b) is in the office (but not both). Both tend to stay in the office when they are in. Now I see Bob going out of the office. What do I believe now?

Since there is at least one possible world μ that implies (a) and ($\neg b$), it is not possible to eliminate it in KM-update and, consequently, the agent cannot belief in ($\neg a$) after the update.

$$\|\varphi\| = \{\neg a \wedge b, a \wedge \neg b\}$$

$$\{a \wedge \neg b\} \subseteq \|\varphi \diamond \neg b\|$$

Ultimate Question

Which conditions do we need to consider to construct a model that represents real world updates?

Ongoing works …

Two ongoing works …

Belief Change: Characterization of KM-Erasure (Extended Abstract)

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Abstract. One of the most important models in the literature of belief change is *update*, defined by Katsuno and Mendelzon in 1992. In their work, KM mentioned *erasure* as the counterpart change of update. However, erasure was only defined at the basic level. In this paper: (1) we complete the axiomatics of erasure a
via the Levi and Harper Identities,
terms of possible worlds.

Keywords: Belief Change · Katsun
· Axiomatic Characterization · possi

- Similarity-based reasoning

Submission dates

- Submission of full papers **(final extension): January 12th, 2**
- Notification of acceptance: February 1st, 2024.
- Camera ready copies due: March 1st, 2024.
- Conference: **May 12th-15th, 2024**.

How to submit

We invite scientific publications up to six pages, which may be submitted in PDF

Two ongoing works …

Constructive Methods for KM-Update and Erasure

Eduardo Fermé[1][0000−0002−9618−2421]

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To be submitted during February … now under revisions and updates.

Conclusions

Conclusions

- KM-Update is one of the most important model of belief change in the literature
- In this talk
  - we revisited its relationship with revision
  - we introduced the notion of iteration of update
  - we introduced the notion of credibility limit for KM-update
  - we analysed some real worlds examples

Papers

- Fermé, Eduardo, and Sara Gonçalves. "On the logic of theory change iteration of KM-update." *International Journal of Approximate Reasoning* 162 (2023): 109005.
- Fermé, Eduardo, Sébastien Konieczny, Ramón Pino Pérez, and Nicolas Schwind. "Credible Models of Belief Update." In Proceedings of the International Conference on Principles of Knowledge Representation and Reasoning, vol. 19, no. 1, pp. 252-261. 2023.
- Fermé, Eduardo , Gabriele Kern Isberner, Tommie Meyer and Abhaya Nayak. "Revision by scenarios: An Unified model for Revision and Update. Ongoing manuscript.

Thank you!

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