Nonmonotonic Reasoning with Infeasible Worlds

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(based on joint work with Christoph Beierle, Gabriele Kern-Isberner, Thomas Meyer)

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Introduction

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- What can weakly consistent belief bases encode that strongly consistent belief can't?
- How can we extend inference operators that assume strongly consistent belief bases to also reason from weakly consistent belief bases?
  - System W $\rightarrow$ system W$^+$
  - c-Inference $\rightarrow$ extended c-inference

Overview

1. Background inductive inference operators
2. Strong and weak consistency
3. Reasoning with infeasible worlds
4. Introduce system W^+
5. Introduce extended c-inference

Conditional Logic

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Conditionals $(B|A)$ with A, B prop. formulas over some finite signature Σ;

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Three valued semantic [deFinetti 1937]:

- $(B|A)$ verified by ω if $\omega \models AB$
- $(B|A)$ falsified by ω if $\omega \models A\overline{B}$
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Example (belief base)

$\Delta = \{(b|p), (f|b), (\neg f|p)\}$ "Penguin triangle"

Ranking Functions as Models for Conditionals

Definition (Ranking function [Spohn 1988])

Function $\kappa : \Omega \to \mathbb{N}_0 \cup \{\infty\}$ such that $\kappa^{-1}(0) \neq \emptyset$.

Intuition: more plausible worlds have lower ranks.
Worlds with rank ∞ are completely infeasible.

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Example (ranking function)

ω	pbf	$pb\overline{f}$	$p\overline{b}f$	$p\overline{bf}$	$\overline{p}bf$	$\overline{p}b\overline{f}$	$\overline{pb}f$	$\overline{p}\overline{bf}$
$\kappa(\omega)$	2	1	∞	∞	0	1	0	0

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Inductive inference operator [Kern-Isberner, Beierle, Brewka 2020]

A mapping $C : \Delta \mapsto \mathrel{|\!\sim}_\Delta$ that maps each belief base to an inference relation such that

(DI) if $(B|A) \in \Delta$ then $A \mathrel{|\!\sim}_\Delta B$ and

(TV) if $\Delta = \emptyset$ and $A \mathrel{|\!\sim}_\Delta B$ then $A \models B$.

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Examples:

- p-entailment [Kraus, Lehmann, Magidor 1990] / system P [Adams 1965]
- system Z [Pearl 1990] / rational closure [Lehmann 1989]
- (skeptical) c-inference [Beierle, Eichhorn, Kern-Isberner 2016]

Strongly and Weakly Consistent Belief Bases

Definition (strongly consistent)

Δ is **strongly** consistent if there is a κ with $\kappa \models \Delta$ and $\kappa^{-1}(\infty) = \emptyset$.

Definition (weakly consistent)

Δ is **weakly** consistent if there is a κ with $\kappa \models \Delta$.

(Note: Δ is weakly consistent iff $\top \not\mid\!\sim^p_\Delta \bot$.)

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Strong consistency is used, e.g., in [Goldszmidt Pearl 1996]
Weak consistency is used, e.g., in [Giordano Gliozzi 2015], [Casini, Meyer, Varzinczak 2019]

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What is the difference between strong and weak consistency?

Consistency of Belief Bases

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there is a κ with $\kappa \models \Delta$ and $\kappa^{-1}(\infty) = \emptyset$.

Example

$\Delta = \{(B|A), (\overline{B}|A)\}$ is inconsistent.

Only defeasible beliefs,
every formula can be somewhat plausible.

Δ is **weakly** consistent if
there is a κ with $\kappa \models \Delta$.

Example

$\Delta = \{(B|A), (\overline{B}|A)\}$ is consistent.

But it requires that A is unfeasible under any condition.

Might contain "strict" constraints.

Weakly Consistent Belief Bases and "Strict" Beliefs

"Strict" beliefs / hard constraints can be expressed by defeasible conditionals [KLM90]:

$$\text{strictly } A \text{ corresponds to } (\bot|\overline{A})$$
$$\overline{A} \mathrel{|\!\sim} \bot$$

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Observation: Belief bases that are not strongly consistent contain strict beliefs.

Proposition

For a belief base Δ that is not strongly consistent there is at least on world ω with $\omega \mathrel{|\!\sim}^{p}_{\Delta} \bot$.

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Proposition

For a belief base Δ that is not strongly consistent there is at least on world ω with $\omega \mathrel{|\!\sim}^{p}_{\Delta} \bot$.

$\rightarrow$ Every preferential inference relation
satisfying (DI) wrt. a Δ that is not strongly consistent contains strict beliefs.

Extending Inference Operators for Weakly Consistent Belief Bases

Observation

Some inference operators are defined for all belief bases (p-entailment, rational closure, ...)
Some inference operators are only defined for strongly consistent belief bases (system W, c-inf.)

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Fill this gap by extending inference operators to handle infeasible worlds;
here done for

- **system W** (cf. [Haldimann, Beierle, Kern-Isberner, Meyer 2023]) and
- **c-inference** (cf. [Haldimann, Beierle, Kern-Isberner 2023]).

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System W is an inductive inference operator [Komo, Beierle 2020; 2022]

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Now: extend system W to cover *all* belief bases $\rightarrow$ **System W$^+$**

Tolerance Partition

Inclusion maximal tolerance partition [Pearl 1990; Goldszmidt, Pearl 1996]

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Example

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Preferred structure on worlds

ξ^j, ξ, preferred structure $<^{\mathsf{w}+}_{\Delta}$ on worlds

$\Delta = \{r_i = (B_i|A_i) \mid i \in \{1, \ldots, n\}\}$ with $EZP(\Delta) = (\Delta^1, \ldots, \Delta^k, \Delta^\infty)$.

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Preferred structure on worlds: $(\Omega^{feas}, <^{\mathsf{w}+}_{\Delta})$
$<^{\mathsf{w}+}_{\Delta} \subseteq \Omega^{feas} \times \Omega^{feas}$ defined by, for any $\omega, \omega' \in \Omega$,

$$\omega <^{\mathsf{w}+}_{\Delta} \omega' \ \text{ iff } \ \text{there exists } m \in \{0, \ldots, k\} \text{ such that}$$
$$\xi^i(\omega) = \xi^i(\omega') \quad \forall i \in \{m+1, \ldots, k\}, \text{ and}$$
$$\xi^m(\omega) \subsetneqq \xi^m(\omega').$$

System W^+ – Definition

System W^+, $\mid\!\sim^{\mathsf{w}+}_{\Delta}$

$$A \mid\!\sim^{\mathsf{w}+}_{\Delta} B$$

if for every feasible $\omega' \in \Omega_{A\overline{B}}$ there is a feasible $\omega \in \Omega_{AB}$ such that $\omega <^{\mathsf{w}+}_{\Delta} \omega'$.

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$\rightarrow$ Especially $A \mid\!\sim^{\mathsf{w}+}_{\Delta} \perp$.
This introduces strict beliefs into the induced inference relation.

System W^{+} – Example

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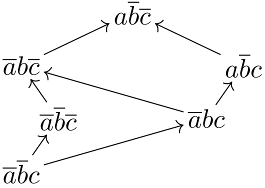

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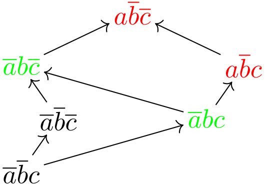

Entailment: e.g. $\overline{a}b \vee a\overline{b} \mathrel{|\!\!\sim}^{\mathsf{w}+}_{\Delta} \overline{a}b$

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- ✓ Extended c-Inference satisfies $A \mathrel{|\!\!\sim}^{ec}_{\Delta} \bot$ iff $A \mathrel{|\!\!\sim}^{p}_{\Delta} \bot$.
  (Classic Preservation)
- ☺ ✓ System W$^+$ complies with syntax splitting.

SPOs on Worlds as a Model for Conditionals

Definition (SPO on worlds)

An *SPO on worlds* is a tuple $(\Omega^{feas}, \prec)$ consisting of

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Lifted to formulas:

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- $A \prec B$ iff for every $\omega' \in \Omega_B \cap \Omega^{feas}$ there is an $\omega \in \Omega_A \cap \Omega^{feas}$ such that $\omega \prec \omega'$.

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As a model for conditionals:

$$(\Omega^{feas}, \prec) \models (B|A), \quad \begin{array}{l} \text{if either } A\overline{B} \text{ is infeasible} \\ \text{or } AB \text{ and } A\overline{B} \text{ are both feasible and } AB \prec A\overline{B}. \end{array}$$

Extended c-Inference

Let $\Delta = \{(B_1|A_1), \ldots, (B_n|A_n)\}$ over Σ.

Definition (c-representation [Kern-Isberner 2001; 2004])

A *c-representation* of Δ is a ranking function $\kappa_{\vec{\eta}}$ constructed from impacts $\vec{\eta} = (\eta_1, \ldots, \eta_n)$ with $\eta_i \in \mathbb{N}_0$ assigned to each conditional $(B_i|A_i)$ such that $\kappa_{\vec{\eta}}$ accepts Δ and is given by:

$$\kappa_{\vec{\eta}}(\omega) = \sum_{1 \leqslant i \leqslant n;\ \omega \models A_i \overline{B}_i} \eta_i .$$

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Extended c-Representations: Example

Let $\Sigma = \{b, p, f\}$ and $\Delta = \{(b|p), (f|b), (\overline{b}|p)\}$.

ω	$(b\|p)$	$(f\|b)$	$(\overline{b}\|p)$	impact on ω
bpf	v	v	f	η_3
$bp\overline{f}$	v	f	f	$\eta_2 + \eta_3$
$b\overline{p}f$	–	v	–	0
$b\overline{p}\overline{f}$	–	f	–	η_2
$\overline{b}pf$	f	–	v	η_1
$\overline{b}p\overline{f}$	f	–	v	η_1
$\overline{b}\overline{p}f$	–	–	–	0
$\overline{b}\overline{p}\overline{f}$	–	–	–	0

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ω	$(b\|p)$	$(f\|b)$	$(\overline{b}\|p)$	impact on ω	$\kappa_{\vec{\eta}}(\omega)$
bpf	v	v	f	η_3	∞
$bp\overline{f}$	v	f	f	$\eta_2 + \eta_3$	∞
$b\overline{p}f$	–	v	–	0	0
$b\overline{p}\overline{f}$	–	f	–	η_2	1
$\overline{b}pf$	f	–	v	η_1	∞
$\overline{b}p\overline{f}$	f	–	v	η_1	∞
$\overline{b}\overline{p}f$	–	–	–	0	0
$\overline{b}\overline{p}\overline{f}$	–	–	–	0	0
impacts:	η_1	η_2	η_3		
$\vec{\eta}$	∞	1	∞		

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Let Δ be strongly consistent.
Every c-representation $\kappa_{\vec{\eta}}$ of Δ is an extended c-representation of Δ.

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Proposition (Classic Preservation for ranking functions)

Let Δ be weakly consistent.
There is an extended c-representation $\kappa_{\vec{\eta}}$ of Δ such that $\kappa_{\vec{\eta}}(\omega) < \infty$ iff $\omega \not\models^p_\Delta \bot$.

Extended c-Inference

Definition (c-inference, $\mathrel{|\!\sim}^c_\Delta$ [Beierle, Eichhorn, Kern-Isberner 2016])

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Extended c-Inference: Example

Let $\Sigma = \{b, p, f\}$ and $\Delta = \{(b|p), (f|b), (\overline{b}|p)\}$.

ω	$(b\|p)$	$(f\|b)$	$(\overline{b}\|p)$	impact on ω
bpf	v	v	f	η_3
$bp\overline{f}$	v	f	f	$\eta_2 + \eta_3$
$b\overline{p}f$	–	v	–	0
$b\overline{p}\overline{f}$	–	f	–	η_2
$\overline{b}pf$	f	–	v	η_1
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We can see that, e.g., $\overline{p}\overline{f} \mid\!\sim^{ec}_{\Delta} \overline{b}$.

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- ✓ *Extended c-Inference satisfies $A \mathrel{|\!\sim}^{ec}_{\Delta} \bot$ iff $A \mathrel{|\!\sim}^{p}_{\Delta} \bot$. (Classic Preservation)*
- ☺ ✓ *Extended c-inference complies with syntax splitting.*

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Thank you for your attention.