Inductive reasoning, conditionals, and belief revision

Part I – Basics and the general framework

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Overview of this talk – Part I

- Introduction and overview

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- Basics on epistemic states and conditionals

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Traditional view of inductive reasoning

- In an abstract way, inductive reasoning means to generate generic knowledge from examples or observations.

Example

Fifi is a dog and barks
Lassie is a dog and barks
Bert is a dog and barks
Lady is a dog and barks
Bella is a dog and barks

(All) Dogs bark.

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- Inductive reasoning has long been understood as being basically probabilistic, and has been investigated mainly with Bayesian methods; Carnap's inductive logic is maybe the most famous representative here.

A novel vision of inductive reasoning

Inductive reasoning . . .

. . . should be able to "generate" new (generic) beliefs from given beliefs and ideally, complete the beliefs of a human being as far as possible. [GKI, Spohn - JAL 2024]

Partial Beliefs
Belief/Knowledge Base

$\xrightarrow[\text{(principled)}]{\text{Induction}}$

(sets of)
"New" Inferred Beliefs,
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- Probabilistic Principle of maximum entropy (MaxEnt)
  Input: set of probabilistic conditionals
  Output: probability distribution

Conditionals aka defeasible rules

Defeasible rules establish an uncertain, defeasible connection between antecedent A and consequent B of a rule and can be (logically) implemented by conditionals

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- Conditionals implement nonmonotonic inferences via "$(B|A)$ is accepted iff $A \mathrel{|\!\sim} B$ holds".
- Conditionals occur in different shapes in many approaches (e.g., as conditional probabilities in Bayesian approaches),
- Conditionals seem to be similar to classical (material) implications "If A then (definitely) B", but are substantially different!

  Indeed, many fallacies observed when applying classical logic to uncertain domains are caused by mixing up implications and conditionals!

Conditionals and implications – example

Christmas on the northern hemisphere

- If Christmas were in summer, there would be no snow at Christmas.
- If Christmas were in summer, there would be no Christmas gifts.
- If Christmas were in summer, there would be no gravitation.

All these statements are logically true, when understood as (material) implications (because Christmas is in winter on the northern hemisphere, hence the antecedent is false!).

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  downright nonsense!

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However, understood as conditionals, crucial differences appear!

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A conditional leaves more semantical room for modelling acceptance in case its confirmation $A \wedge B$ is more plausible than its refutation $A \wedge \neg B$.

Conditional acceptance and preferential entailment $\mid\!\sim_{\prec}$ [Makinson 89]

Let $\prec$ be a (well-behaved) relation on models (expressing , e.g., plausibility via a total preorder $\preceq$).

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iff in the most plausible models of A (wrt $\prec$), B holds also.

$\mid\!\sim_{\prec}$ is a semantic-based nonmonotonic inference relation that is encoded by conditionals on the syntax level.

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- Well-behaved relations on possible worlds expressing (e.g.) plausibility provide semantics to both conditionals and nonmonotonic inference relations.
- Note that plausibility relations are similar to, but significantly weaker than probabilities.

Ranking functions and conditionals

A particular useful implementation of a plausibility relation:

Ordinal conditional functions (OCF, ranking functions[1]) [Spohn 1988]

$\kappa : \Omega \to \mathbb{N}(\cup\{\infty\})$ (Ω set of possible worlds, $\kappa^{-1}(0) \neq \emptyset$)

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Validating conditionals

$$\kappa \models (B|A) \text{ iff } \kappa(AB) < \kappa(A\overline{B}) \text{ iff } A \mid\!\sim_\kappa B$$

- κ accepts a conditional $(B|A)$
  - iff its verification AB is more plausible than its falsification $A\overline{B}$
  - iff from A, defeasibly infer B (based on κ).

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Ranking functions – example

Example (ranked flyers)

$\kappa(\omega) = 4$	$p\bar{b}\,f$
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Ranking functions make conditional and nonmonotonic reasoning particularly easy!

Motivating example

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Now a young girl being addicted to alcohol visits the helpcenter, asking the psychologist for help.

Will the psychologist expect her to be also addicted to drugs?

Motivating example (cont'd) and inductive perspective

Now, the psychologist will change his job, he will be working in a clinic in which exclusively people being addicted to alcohol and/or drugs are treated. He knows that in this clinic, the rate of people being addicted to alcohol, but also addicted to drugs is higher than usual.

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→ For inductive reasoning, we need to be able to

- reason from conditional belief bases
- in dynamic environments, i.e., also involving belief revision.

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- Beliefs are expressed by conditionals $(B|A)$ in the first place, where plain beliefs A are also covered by identifying the plausible belief A with the conditional $(A|\top)$, where $\top$ is a tautology: $A \equiv (A|\top)$.
- This joint framework of inductive reasoning and belief revision is exemplified
    - in probabilistics, via the principles of optimum entropy;
    - for qualitative reasoning, via ranking functions and c-revisions.

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$\mathit{Mod}(A)$	set of models of A

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$A \Rightarrow B$	$\equiv \neg A \vee B$ material implication

Semantics:

Ω	set of interpretations/models/possible worlds over Σ
$\omega \models A$	ω is a model of $A (\in \mathcal{L})$
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Basics of propositional logic

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$Cn(A)$	$= \{B \in \mathcal{L} \mid A \models B\}$ classical consequence operator

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  - In probabilistics: $(\mathcal{L} \mid \mathcal{L})^* = (\mathcal{L} \mid \mathcal{L})^{prob} = \{(B|A)[x] \mid A, B \in \mathcal{L}, x \in [0,1]\}$
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- Entailment/Inference: A satisfaction relation $\models$ is given between epistemic states and conditionals: $\Psi \models (B|A)^*$ means that $(B|A)^*$ is accepted in Ψ, where acceptance is defined suitably.
  - For a probability distribution P, $P \models (B|A)[x]$ iff $P(A) > 0$ and $P(B|A) = x$.
  - For a ranking function κ, $\kappa \models (B|A)$ iff $\kappa(AB) < \kappa(A\overline{B})$.

Meta-structures associated with epistemic states

Examples of meta-structures $\preceq_\Psi$ to be associated with epistemic states for (inductive) reasoning and revision:

- Total preorders
- Ranking functions (OCFs)
- Possibility distributions
- Modal logic frameworks
- Probabilities

Rich Meta-Structures

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Some of these features are also provided by ranking functions (OCFs, [Spohn, 1988]) and possibility distributions.

Epistemic states and conditionals

Basic idea of conditional acceptance

A conditional $(B|A)$ is accepted in an epistemic state if its *verification* AB is deemed to be more plausible, probable etc than its *falsification* $A\overline{B}$.

By considering A and B resp. A and $\overline{B}$ jointly when assessing plausibility, probability, and the like, truth functionality is lost, but cognitive adequacy is gained because for conditionals, humans would expect a meaningful connection between antecedent and consequent (in contrast to material implications).

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Recall: Accepting a conditional $(B|A)$ is basically the same as drawing the nonmonotonic inference $A \mid\!\sim B$ via preferential entailment.

Conditional belief bases

Let $\mathcal{E}^* = \mathcal{E}^*_\Sigma$ denote the set of all such epistemic states based on $(\mathcal{L} \mid \mathcal{L})^*$, $\mathcal{L} = \mathcal{L}(\Sigma)$.

Epistemic states are (epistemic) models of conditional belief bases, i.e., (finite) sets of conditionals $\Delta \subseteq (\mathcal{L} \mid \mathcal{L})^*$:

$$Mod^*(\Delta) = \{\Psi \in \mathcal{E}^* \mid \Psi \models \Delta\}.$$

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$\Delta \subseteq (\mathcal{L} \mid \mathcal{L})^*$ is consistent iff $\mathit{Mod}^*(\Delta) \neq \emptyset$, i.e. iff there is an epistemic state which is a model of Δ.

Example

The belief base $\Delta_{psych} = \{(\overline{d}|a), (\overline{a}|d), (\overline{a}|y), (d|y), (\overline{d}|\overline{y}), (a|\overline{y})\}$ can be interpreted by epistemic states that are equipped with a total preorder, or with a ranking function [Spohn, 1988]; it is consistent.

Inductive reasoning from conditional belief bases

Choosing a "best" model Ψ^* of Δ allows for inductive reasoning from Δ via

$$A \mid\!\sim^{\Delta}_{\Psi^*} B \quad \text{iff} \quad \Psi^* \models (B|A).$$

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We need to talk about belief revision ...

The core ideas of AGM theory

The main approach to belief revision in KR is the AGM theory [Alchourron, Gärdenfors, Makinson 1985], dealing with revising a belief set K by new propositional information A:

$$K * A.$$

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- In case of consistency, belief change should be performed via expansion.
- The result of belief change should depend only upon the semantical content of the new information.

200 years before . . .

Considering the task of belief change is not new: About 200 years before AGM theory, Bayes came up with his famous rule in probabilistics:

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Actually, Bayesian conditioning fulfills the core ideas of AGM theory, but obviously, the contexts of the theories (changing a code of law for AGM vs. random experiments and chances – e.g., in gambling – for Bayes) seemed to be too diverse to realize a strong connection.

The general task of belief change

However, from a formal resp. epistemic point of view, the tasks are similar if not identical:

General task of belief change

Given some (prior) epistemic state Ψ and some new information I, change beliefs rationally by applying a change operator $*$ to obtain a (posterior) epistemic state Ψ':

$$\Psi * I = \Psi'$$

AGM	: $\Psi = K$	set of propositional beliefs
Bayes	: $\Psi = P$	probability distribution
both	: $I = A$	propositional belief

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- One-step revision: AGM belief revision does not consider changes of epistemic states nor revision strategies
  $\rightarrow$ Iterated revision
- New information: Only one proposition – what about sets of propositions, conditional statements, sets of conditionals?
  $\rightarrow$ Conditional and multiple belief revision

Iterated belief revision

Iterating belief revision means handling tasks of the form

$$((\Psi * A) * B) * C$$

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In iterative belief revision, the AGM principle of minimal change is replaced or complemented by a principle of conditional preservation [Darwiche & Pearl, AIJ 1997].

Iterated belief revision and AGM

Crucial result [Katsuno & Katsuno, 1991] for iterating AGM revision, resp. lifting AGM to the level of epistemic states:

All AGM postulates for revision can be fulfilled iff there is a total preorder $\preceq_K$ on possible worlds s.t. $Bel(\preceq_K) = K$ and

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- Epistemic states for AGM revision need to be equipped with total preorders (at least) ($\rightarrow$ meta-structures).
- Close connection to preferential entailment: $\min_{\prec} Mod(A) \subseteq Mod(B)$.

Fundamental equivalences among induction, revision, and conditionals

Fundamental connection between

- epistemic states $\Psi \in \mathcal{E}^*$,
- $\preceq_\Psi$ suitable relation expressing plausibility, probability etc,
- conditionals,
- inductive inference relation $\mid\!\sim_\Psi$ based on Ψ, and
- epistemic (or iterative) revision operator $*$ in the sense of [D&P 1997]:

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- Both epistemic states and conditionals are carriers of strategic information that become effective for reasoning and revision.
- More generally, we assume that $*$ can also be used for epistemic, conditional revision $\Psi * \Delta \in \mathcal{E}^*$ such that $\Psi * \Delta \models \Delta$ holds [2].

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Overview of this talk – Part I

- Introduction and overview
- Basics on epistemic states and conditionals
- Induction and revision – a general framework

Inductive reasoning from epistemic states

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- $\Delta =$ set of probabilistic conditionals; *ind* can be the MaxEnt principle

Inductive reasoning from Δ is then implemented by reasoning from $\Psi_\Delta = \mathit{ind}(\Delta)$ via the fundamental equivalences above and the conditionals being accepted in Ψ.

Inductive reasoning from epistemic states – example

$\Delta_{psych} = \{(\bar{d}|a), (\overline{a}|d), (\overline{a}|y), (d|y), (\bar{d}|\overline{y}), (a|\overline{y})\}$

- qualitative framework: Choose system Z or c-representations to select a "best" model κ_{psych};

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But that's not the end of the story . . .

Receiving new information $\mathcal{I}$

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$\rightarrow$ Belief revision, returning a new epistemic state Ψ' after revising Ψ by $\mathcal{I}$:

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Challenges:

- How do *ind* and $*$ interact?
- What roles do Ψ_Δ, Δ and $\mathcal{I}$ play in this scenario?

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Note that the revision operator $*$ is used in quite a generic sense (technical details to be elaborated in the following).

Different revision scenarios

The agent's new epistemic state Ψ' should arise from the adaptation of background beliefs Ψ_Δ to new contextual information $\mathcal{I}$, and this process should be iterative, i.e., also being able to take further information $\mathcal{I}'$ into account:

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- First case: $\mathcal{I}'$ refers to the same context as $\mathcal{I}$, narrowing the context (conservative revision, epistemic expansion). In this case, $\mathcal{I}$ and $\mathcal{I}'$ should be considered on the same level, and we propose $\Psi_\Delta * (\mathcal{I} \cup \mathcal{I}')$.

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- Second case: $\mathcal{I}'$ is information on a new, shifted context for which, however, $\mathcal{I}$ is still relevant (update). Then we propose $(\Psi_\Delta * \mathcal{I}) * \mathcal{I}'$ with two revision operators of the same kind.

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- Second case: $\mathcal{I}'$ is information on a new, shifted context for which, however, $\mathcal{I}$ is still relevant (update). Then we propose $(\Psi_\Delta * \mathcal{I}) * \mathcal{I}'$ with two revision operators of the same kind.
- Third case: $\mathcal{I}'$ affects background beliefs (learning). If $\mathcal{I}'$ is fully compatible with Δ, we propose $\mathit{ind}(\Delta \cup \mathcal{I}') * \mathcal{I}$, otherwise, we propose $(\mathit{ind}(\Delta) * \mathcal{I}') * \mathcal{I}$.

Revision scenarios – example

$\Delta_{psych} = \{(\overline{d}|a), (\overline{a}|d), (\overline{a}|y), (d|y), (\overline{d}|\overline{y}), (a|\overline{y})\}$

Now a young girl being addicted to alcohol visits the helpcenter, ...

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Now a young girl being addicted to alcohol visits the helpcenter, ... Later on, the psychologist changes his job ...

Revision scenarios – example

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- Update: New job/context – only people being addicted to alcohol and/or drugs, and being addicted to drugs in the context of alcohol abuse is (more) plausible

$$\Psi_{psych} * \{(a \vee d|\top), (d|a)\}.$$

Revision scenarios – example (cont'd)

- Learning: While treating the young girl being addicted to alcohol (in his old job), the psychologist reads in a medical journal that for young people being addicted to alcohol, it is (more) plausible to also being addicted to drugs

$$\mathit{ind}(\Delta_{psych} \cup \{(d|ya)\}) * (y \wedge a).$$

Induction and belief revision

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$$\mathit{ind} = \mathit{ind}_{\Psi_u} \quad \text{and} \quad \Psi_\Delta = \mathit{ind}(\Delta) = \Psi_u * \Delta.$$

$\rightarrow$ thorough implementation of induction via belief revision.

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  which helps analyzing it.
- Quality criteria from belief revision (postulates, lots of them are available) can be useful to classify approaches to induction.

We need examples here $\rightarrow$ Part II.